NASA CONTRACTOR REPORT

NASA CR-2356

NASA CR-2356

ANALYSIS OF LAMINAR FLOW BETWEEN STATIONARY AND ROTATING DISKS WITH INFLOW

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • FEBRUARY 1974

1. Report No. NASA CR-2356	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle ANALYSIS OF LAMINAR FLOW BETWEEN STATIONARY AND ROTATING DISKS WITH INFLOW		5. Report Date February 1974
		6. Performing Organization Code
7. Author(s) Upendra Rohatgi and Eli Reshotko		8. Performing Organization Report No. FTAS/TR 73-85
		10. Work Unit No.
9. Performing Organization Name and Address Case Western Reserve University 2040 Adelbert Road Cleveland, Ohio 44106		11. Contract or Grant No. NGL 36-003-139
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Final Report. Project Manager, Werner R. Britsch, Fluid System Components Division, NASA Lewis Research Center, Cleveland, Ohio

16. Abstract

An analysis has been made of the laminar flow between a rotating and a stationary disk with inflow. Solutions to the dimensionless governing equations are sought by expanding each of the velocity components in powers of inverse radius. The equations to leading order are those for the configuration with no inflow. The subsequent orders yield sets of linear ordinary differential equations. Solutions are obtained for the first two of these subsequent orders. The solutions indicate that inflow tends to increase the magnitude of the azimuthal velocity in the flow between the two disks and to decrease the torque on the rotating disk. For Prandtl number one, an energy integral is obtained which relates the temperature distribution to the velocity distribution for all Reynolds numbers and therefore eliminates the need for separate solution of the energy equation. The importance and use of the presented solutions for design purposes are discussed and an example is given.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Rotating disks Turbulent boundary layers Pumps	Unclassified - unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	117	Domestic, \$4.25 Foreign, \$6.75

* For sale by the National Technical Information Service, Springfield, Virginia 22151

NASA-C-168 (Rev. 6-71)

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TABLE OF CONTENTS

LIST OF FIGURES

FIGURE		PAGE

LIST OF TABLES

LIST OF SYMBOLS

C	specific heat
F_n	n^{th} term in the modified expansion ofstream function
f_n	n^{th} term in the expansion of stream function
G_n	n^{th} term in the modified expansion of azimutal velocity
g_n	n^{th} term in the expansion of azimuthal velocity
H	total enthalpy
H_n	n^{th} term in the expansion of total enthalpy
h_n	n^{th} term in the expansion of pressure
h	integral constant in the expansion of pressure
PL	power loss factor
P	pressure
Q	leakage flow
R	reference radius
R_o	external radius of pump
r_o	internal radius of pump
r	radial coordinate
S	gap width between disks
T	temperature
u	radial velocity
V	azimuthal velocity
W	axial velocity
Z	axial coordinate
Z_1	distance of match point from the rotating disk
β	ratio of core angular velocity to disk angular velocity

μ viscosity

ν kinematic viscosity

ρ density

Ω disk angular velocity

SUBSCRIPTS

0,1,2 terms in the expansions

D reference to disk

h solution of homogeneous equation

Imp impeller

L left side

p particular solution

R right side

ref reference quantity

Prime denotes differentiation with respect to Z and dimensional quantities are barred.

I. INTRODUCTION

The demanding requirements for pump performance and efficiency in recent aeronautics and space applications have highlighted the need for more accurate knowledge of flow conditions in centrifugal and disk pumps. Specifically there is need for better estimation of shear stresses and pressure distributions for the flow between impeller disk and housing in order to obtain better estimates of power losses and axial thrust. Knowledge of axial thrust is required in designing bearings for the impeller. Cooper [1] has in fact shown that for small centrifugal pumps having specific speeds under 500, the retarding torque due to impeller rotation ("disk friction") is by far the largest source of power loss in such a pump.

Due to the imperfection of shaft seals, there tends to be leakage from the exit of the impeller through the region between the impeller and housing, and out past the shaft seals. This leakage flow may have significant effect on the disk friction of the impeller and on the pressure distribution in the region between impeller and housing. It is exactly these effects of leakage that are investigated in the present study.

More specifically, the flow problem treated herein is that between two infinite disks, one of which is stationary and the other rotating with inflow through the narrow gap between them. This

inflow is assumed to leave at the axis of rotation. The assumption of infinite disks precludes the consideration of any end effects.

The basic flow configuration is as follows: The rotating disk imparts rotation to the fluid leading to a distribution of angular velocity from the rotating disk value at the disk to zero at the stationary wall. The pressure in the narrow gap is centrifugally generated and its magnitude depends on the effective average angular velocity of the flow field. The pressure tends to be a function of radius only and independent of axial position in the narrow gap. The angular velocity of the fluid near the rotating disk is above the effective average and hence the fluid near the rotating disk is thrown radially outward. On the other hand, fluid close to the stationary wall has a smaller angular velocity than the effective average so that it flows radially inward. These secondary flows are modified somewhat by the inflow due to leakage.

This inflow has significant angular momentum gained in the impeller. As it flows inward it imparts its angular momentum to the surrounding fluid which in turn rotates faster, so that the effect of inflow is to change the velocity distributions in the gap between the rotating disk and the wall. This azimuthal speeding up of the flow between the two disks tends to reduce the shear stresses on the rotating disk and increase the shear stresses on the wall. It is the objective of this study to assess quantitiatively this effect of leakage as well as the influence of leakage on the velocity and temperature distributions in the fluid.

The flow between the disk and the wall is laminar in the vicinity of the axis of rotation and then becomes turbulent beyond the radius where the flow undergoes transition. In this study only the laminar region is investigated. Although the laminar flow analysis applies directly to only a portion of the flow system, it will suffice to demonstrate the effects of inflow and also provide the base solution for constructing solutions in the transition and turbulent regimes.

Previous Investigations

There have been numerous studies of the flow between two rotating disks but comparatively few of them have included inflow. The studies with inflow have been either by momentum integral methods or by approximate solution of the Navier-Stokes equations.

The earliest study of the problem with inflow was undertaken by Jimbo [2] for turbulent flow using a momentum integral method. Jimbo ignored the axial variation of azimuthal velocity as well as the effect of inflow on the secondary flow pattern. He estimated the radial pressure gradient with the aid of friction factors obtained from Schultz-Grunow [3]. Jimbo's calculated values show the trends observed in his experiments and led him to conclude that: (a) the variation of tangential velocity with radius does not change with clearance; (b) the pressure distribution with radius for a given leakage is independent of clearance, and (c) the power loss due to skin friction decreases as the leakage flow rate increases.

This momentum integral approach is fast but somewhat suspect due to the lack of flow details in the gap.

Among the studies based on approximate solution of the Navier-Stokes equations, Makay and Trumpler [4] have investigated the present problem for laminar flow with leakage, emphasizing the departure from the solution for Reynolds number zero. They formed a non-linear integral equation and solved it by a Fredholm integral technique which strictly speaking is applicable to linear problems. The solution to the non-linear inertia terms as known from the prior iteration. They solved for a rather limited range of gap width, namely up to $S = 3.578$ (or gap-width Reynolds number of 3.578).

The class of solution procedure leading to the one used herein is that of linearization. An early example of such a procedure is that of Soo [5]. A meridional stream function is constructed adding an inflow term to the expression for streamfunction without inflow. This is substituted into the boundary layer equations and solutions obtained for small inflow (or outflow) by expansion about the conditions for zero gap-width Reynolds number. Soo's work although deficient because of assuming equal shear stresses on both the rotating and stationary disks, sheds light on the recirculation of flow between the disks and on the effects of inertia.

The linearization scheme of the present paper is essentially due to an expansion of the meridional stream function in the form

$\psi \sim \sum_n \left(\frac{1}{r}\right)^n f_n(z)$. This approach was first used by Savage [6] to solve the problem of inflow between two stationary disks. Subsequently Peube and Kreith [7] used this expansion procedure for the case in which both disks are rotating at the same angular velocity. Their solution indicated the explicit appearance of the amount of inflow in coefficients of up to the second order terms. The higher order functions of z are more complicated in that the flow rate appears implicitly; but fortunately, the higher order terms tend to be small. Kreith and Viviand [8], using this procedure, found solutions with inflow between a rotating and stationary disk, but because they also used series expansion about the solution for zero gap-width Reynolds number, their solutions are limited to gap-width Reynolds numbers of the order of one.

Present Study

The procedure of the present study of inflow between a rotating and a stationary disk is that of an expansion about the exact similarity solution without inflow of Reshotko and Rosenthal [9] valid for arbitrary gap-width Reynolds number. The zero order system of equations [9] in this instance is non-linear but the equations of the subsequent orders are linear. The asymptotic expansion scheme of Peube and Kreith [7] is used and as with Peube and Kreith [7] and Kreith and Viviand [8], the inflow rate appears explicitly as a coefficient in the first two perturbation orders while it appears implicitly in the differential equation for higher perturbation

orders. The solution herein is carried only through the first two orders of perturbation. The zeroth order solution procedure used herein is that developed by Cooper [10] for the laminar region in his concurrent study of the laminar, transitional and turbulent flow between rotating and stationary disks without inflow.

The present analysis also includes study of the heat generation and temperature distribution between the disks. For Prandtl number one, an energy integral is obtained which relates the temperature distribution to the velocity distribution regardless of Reynolds number very much as Reshotko and Rosenthal [9] have done for the case of zero inflow.

II. GOVERNING EQUATIONS

This problem of rotating disks is best treated by solving governing equations in a cylindrical co-ordinate system $(\bar{r},\theta,\bar{z})$. As shown in figure 1, the disk in the plane $\bar{z}=0$ is rotating with angular velocity Ω , and the gap width between this and the stationary disk is $\bar{S}$. The symbols, $\bar{u}$, $\bar{v}$, $\bar{w}$ represent velocities in radial, azimuthal and axial directions respectively.

Continuity

$$\frac{\partial(\bar{u}\bar{r})}{\partial\bar{r}} + \frac{\partial(\bar{w}\bar{r})}{\partial\bar{z}} = 0 \qquad (2\text{-}1)$$

Momentum

$$\bar{u}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{u}}{\partial\bar{z}} - \frac{\bar{v}^2}{\bar{r}} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial\bar{r}} + \nu\left[\frac{\partial}{\partial\bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial(\bar{u}\bar{r})}{\partial\bar{r}}\right) + \frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right] \qquad (2\text{-}2)$$

$$\bar{u}\frac{\partial\bar{v}}{\partial\bar{r}} + \bar{w}\frac{\partial\bar{v}}{\partial\bar{z}} + \frac{\bar{u}\bar{v}}{\bar{r}} = \nu\left[\frac{\partial}{\partial\bar{r}}\left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}(\bar{v}\bar{r}\right) + \frac{\partial^2\bar{v}}{\partial\bar{z}^2}\right] \qquad (2\text{-}3)$$

$$\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial \bar{z}} + \nu\left[\frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{w}}{\partial \bar{r}}\right) + \frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right] \tag{2-4}$$

Energy Equation

$$\rho C\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{T}}{\partial \bar{z}}\right) = K\left(\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right)$$

$$+ \bar{u}\frac{\partial \bar{P}}{\partial \bar{r}} + \bar{w}\frac{\partial \bar{P}}{\partial \bar{z}} + \mu\left[2\left(\frac{\partial \bar{u}}{\partial \bar{r}}\right)^2 + 2\left(\frac{\bar{u}}{\bar{r}}\right)^2 + 2\left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2\right.$$

$$\left. + \left(\frac{\partial \bar{v}}{\partial \bar{z}}\right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{r}}\right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{r}} - \frac{\bar{v}}{\bar{r}}\right)^2\right] \tag{2-5}$$

Assumptions:

The fluid model chosen is that of an incompressible, constant property, Newtonian fluid. Body forces are neglected. The disks are impermeable, so fluid velocities normal to the disks are zero and the no slip condition is satisfied at the disks. The flow is steady and axisymmetric.

Non-dimensionalization:

The steady, axisymmetric Navier-Stokes equations and the energy equation for a constant property fluid are non-dimensionalized with respect to characteristic reference quantities. The reference quantities chosen (as from Peube-Kreith [7]) are as follows:

$$L_{ref} = \sqrt{\nu/\Omega} \qquad\qquad U_{ref} = \sqrt{\nu\Omega}$$

$$P_{ref} = \mu\Omega \qquad\qquad T_{ref} = T_w$$

So that the non-dimensional equations are

Continuity

$$\frac{\partial(ur)}{\partial r} + \frac{\partial(wr)}{\partial z} = 0$$

Momentum

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial P}{\partial r} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(ur)}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} \tag{2-6}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(vr)}{\partial r}\right) + \frac{\partial^2 v}{\partial z^2} \tag{2-7}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2} \qquad (2\text{-}8)$$

Energy Equation

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{K}{\rho c \nu}\left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + \frac{\nu\Omega}{C T_{ref}}\left(u\frac{\partial p}{\partial r} + w\frac{\partial p}{\partial z}\right) + \frac{\nu\Omega}{C T_{ref}}\left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{u}{r}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)^2 + \left(\frac{\partial v}{\partial r} - \frac{v}{r}\right)^2\right]$$

Groups of flow parameters which occur in the energy equation, can be put in the form of known non-dimensional numbers

$$\frac{\rho c \nu}{K} = \text{Prandtl number}$$

$$\frac{\nu\Omega}{C T_{ref}} = \text{Eckert number}/\text{Reynolds number}$$

Boundary Conditions

The wall and the disk are impermeable and the no slip condition is satisfied on them. The rotating disk is thermally insulated, while the stationary disk is cooled and kept at constant temperature. The boundary conditions accordingly are:

$$u(0) = 0 \qquad u(s) = 0 \tag{2-10}$$

$$W(0) = 0 \qquad W(s) = 0 \tag{2-11}$$

$$V(0) = r \qquad V(s) = 0 \tag{2-12}$$

$$\frac{\partial T}{\partial z}(0) = 0 \qquad T(s) = 1 \tag{2-13}$$

An additional condition comes from the leakage. If $\bar{Q}$ is the amount of leakage, then:

$$\bar{Q} = \int_0^{\bar{s}} 2\pi \bar{r}\, \bar{u}\, d\bar{z}$$

$$= 2\pi r \nu \sqrt{\nu/\Omega} \int_0^{s} u\, dz$$

or

$$\frac{\bar{Q}}{2\pi\nu\sqrt{\nu/\Omega}} = r \int_0^{s} u\, dz$$

With Q ref defined as:

$$Q_{ref} = 2\pi\nu\sqrt{\nu/\Omega}$$

Then the leakage condition is

$$\frac{Q}{\mathcal{R}} = \int_0^S u \, dz \tag{2-14}$$

where $Q = \overline{Q}/Q_{ref}$. For inflow Q is negative.

For constant property flow, the continuity and momentum equations are independent of the energy equation. Once they are solved and the flow field known, the temperature distribution can be determined by solving the energy equation. Accordingly, the flow field will be considered first and the thermal considerations left for later.

III. SOLUTION OF THE FLOW FIELD

The velocity and the pressure distributions are obtained by solving the momentum and continuity equations. Von Karman has shown that the different components of the velocity in laminar flow, generated by a rotating disk are of following form:

$$u = r F'(z)$$

$$v = r G(z) \qquad (3\text{-}1)$$

$$w = -2 F(z)$$

This form of solution transforms the momentum equations into ordinary differential equations and satisfies continuity. The above form has been successfully used by Reshotko & Rosenthal [9] and also by Cooper [10], to find the velocity field between rotating and stationary disks, without inflow. A modified expansion scheme is required, however, to incorporate inflow into the solution. Peube and Kreith [7] have suggested a method of solving the problem with inflow, by expanding the velocity field in terms of $1/r^n$, with relations (3-1) providing the first terms in the expansions. Their expressions for the velocity components are as follows:

$$\left.\begin{aligned} u &= r f_0' + f_1' + f_2'/r + f_3'/r^2 + \cdots + f_n'/r^{n-1} \\ v &= r g_0 + g_1 + g_2/r + g_3/r^2 + \cdots + g_n/r^{n-1} \\ W &= -2f_0 - f_1/r + 2f_3/r^3 + \cdots + (n-1)\, f_n/r^{n+1} \end{aligned}\right\} \quad (3\text{-}2)$$

With the coefficients f_i and g_i assumed to be functions of both Z and Q. After substituting these into the momentum equations, Peube and Kreith [7] observed that for all Q

$$f_1(Z) = f_3(Z) = \cdots = f_{2n+1}(Z) = 0$$

$$g_1(Z) = g_3(Z) = \cdots = g_{2n+1}(Z) = 0$$

They solved the special case of the disks and the enclosed fluid rotating at the same azimuthal velocity. They obtained a set of ordinary differential equations with constant coefficients, which they solved analytically. The form of the solution suggests that it is possible to separate f_i and g_i in terms of Q and functions of Z up to f_4 , g_4 but for higher order equations Q appears as a parameter in them. So solution functions which are independent of Q, can only be obtained up to order $1/r^3$. This expansion was also used by Kreith and Viviand [8] for finding the velocity field between two rotating disks, with Taylor number $(\Omega \bar{s}^2/\nu) < 4$. They have solved for $f_2(Z)$ and $g_2(Z)$ by series expansion. The technique is applicable here as well.

The modified series for the velocity field appropriate to the present problem are therefore:

$$u = r F_o' + (Q/r)\, F_1' + (Q^2/r^3)\, F_2' + \sum_{m=3}^{\infty} \frac{F_m'(Z;Q)}{r^{2m-1}} \qquad (3\text{-}3)$$

$$v = r G_o + (Q/r)\, G_1 + (Q^2/r^3)\, G_2 + \sum_{m=3}^{\infty} \frac{G_m(Z;Q)}{r^{2m-1}} \qquad (3\text{-}4)$$

In which F_o, F_1, F_2, G_o, G_1, G_2 are function of Z alone.

A stream function can be defined which will satisfy the continuity equation.

$$r u = \frac{\partial \psi}{\partial Z}$$

$$r W = -\frac{\partial \psi}{\partial r}$$

so

$$\psi = r^2 F_o + Q F_1 + \left(\frac{Q^2}{r^2}\right) F_2 + \sum_{m=3}^{\infty} \frac{F_m(Z;Q)}{r^{2m-2}}$$

and therefore

$$W = -2F_o + \left(\frac{2Q^2}{r^4}\right) F_2 - 2\sum_{m=3}^{\infty} (m-1)\, r^{-2m} F_m(Z;Q) \qquad (3\text{-}5)$$

Boundary conditions corresponding to these expansions can be found from (2-10, 2-11, 2-12, 2-14):

$$\left.\begin{array}{ll} F_n'(0) = 0 \; ; & n = 0,1,2 \\ F_m'(0,Q) = 0 \; ; & m = 3,4, --- \infty \end{array}\right\} \quad (3\text{-}6)$$

$$\left.\begin{array}{ll} F_n'(s) = 0 \; ; & n = 0,1,2 \\ F_m'(s,Q) = 0 \; ; & m = 3,4, ---- \infty \end{array}\right\} \quad (3\text{-}7)$$

$$\left.\begin{array}{l} G_0(0) = 1; \quad G_n(0) = 0; \quad n = 1,2 \\ \qquad G_m(0,Q) = 0; \; m = 3,4, --- \infty \end{array}\right\} \quad (3\text{-}8)$$

$$\left.\begin{array}{ll} G_n(s) = 0 \; ; & n = 0,1,2 \\ G_m(s,Q) = 0 \; ; & m = 3,4, ------ \infty \end{array}\right\} \quad (3\text{-}9)$$

$$\left.\begin{array}{l} F_0(s) = 0; \quad F_1(0) = \text{arbitrary}; \quad F_2(0) = 0 \\ \qquad F_m(0,Q) = 0; \; m = 3,4, --- \infty \end{array}\right\} \quad (3\text{-}10)$$

$$\left.\begin{array}{l} F_0(s) = 0; \quad F_1(s) = \text{arbitrary}; \quad F_2(s) = 0 \\ \qquad F_m(s,Q) = 0; \; m = 3,4, --- \infty \end{array}\right\} \quad (3\text{-}11)$$

$F_1(o)$ and $F_1(s)$ are not exactly arbitrary their difference being fixed by the leakage condition (2-14)

$$\int_0^s u\,dz = \frac{Q}{\mathcal{R}} \tag{2-14}$$

or

$$\int_0^s F_1'(z)\,dz = 1$$

As only the difference between $F_1(o)$ and $F_1(s)$ is important, it is assumed that

$$F_1(o) = 0 \qquad\qquad F_1(s) = 1 \tag{3-12}$$

The solutions will be obtained through the first three terms of each function (F_o, F_1, F_2; G_o, G_1, G_2), since only these functions are independent of the parameter Q. The validity of this truncation will be discussed with the results. The equations for the F_n's and G_n's are obtained by substituting the expression for the velocities (3-3, 3-4, 3-5) into the momentum equations (2-6, 2-7, 2-8). Thus equations of different orders are obtained by setting the coefficients of different powers of $\mathcal{R}$, equal to zero. For azimuthal equation (2-7):

$n=o \quad (\mathcal{R}^1)$

$$2F_o'G_o - 2G_o'F_o = G_o'' \tag{3-13}$$

$n=1, \quad (r^{-1})$

$$2F_1'G_0 - 2G_1'F_0 = G_1'' \tag{3-14}$$

$n=2, \quad (r^{-3})$

$$2F_2'G_0 + 2F_2G_0' - 2G_2'F_0 - 2G_2F_0' = G_2'' \tag{3-15}$$

The expansion resulting from the radial momentum equation (2-6) can be found in the same way. Again, the radial momentum equation is:

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial p}{\partial r} + \frac{\partial^2 u}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (ur)}{\partial r}\right) \tag{2-6}$$

Substituting the velocity expressions into this equation yields

$$-\frac{\partial p}{\partial r} = \sum_{n=0}^{2}\sum_{i=0}^{n} Q^n r^{-2n+1}\Big[(-2i+1)F_i'F_{n-i}' + 2(n-i-1)F_i''F_{n-i}$$
$$- G_iG_{n-i}\Big] - \sum_{n=0}^{2} Q^n r^{-2n+1} F_n'''$$
$$+ \sum_{n=1}^{2} Q^{n-1} 4(n-1)(2-n) r^{-2n+1} F_{n-1}'$$

This expression suggests that the pressure "p" has following form:

$$P = r^2 h_o(z) + h(z) + Q\,\ln(r)\,h_1(z) + (Q^2/r^2)\,h_2(z) + \cdots$$

The system of equations obtained from the radial momentum equation are:

$n = 0;\ (r)$

$$F_o'^2 - 2F_oF_o'' - G_o^2 = -2h_o + F_o''' \tag{3-16}$$

$n = 1;\ (r^{-1})$

$$-2F_oF_1'' - 2G_oG_1 = -h_1 + F_1''' \tag{3-17}$$

$n = 2;\ (r^{-3})$

$$-F_1'^2 - G_1^2 - 2F_2'F_o' + 2F_2F_o'' - 2G_2G_o - 2F_2''F_o = 2h_2 + F_2''' \tag{3-18}$$

Substituting these expansions for the velocity field and the pressure in the axial momentum equation (2-8) then yields:

(r^2) $$h_o' = 0 \tag{3-19}$$

(r^o) $$h' = -2F_o'' - 4F_oF_o' \tag{3-20}$$

$(\ln r)$ $$h_1' = 0 \tag{3-21}$$

(r^{-2}) $$h_2' = 0 \tag{3-22}$$

Now all the equations are arranged in systems, depending upon the power of radius:

System 1:

$$2F_0'G_0 - 2G_0'F_0 = G_0'' \tag{3-13}$$

$$F_0'^2 - 2F_0F_0'' - G_0^2 = -2h_0 + F_0''' \tag{3-16}$$

$$h_0' = 0 \tag{3-19}$$

$$h' = -2F_0'' - 4F_0F_0' \tag{3-21}$$

System 2:

$$2F_1'G_0 - 2F_0G_1' = G_1'' \tag{3-14}$$

$$-2F_0F_1'' - 2G_0G_1' = -h_1 + F_1''' \tag{3-17}$$

$$h_1' = 0 \tag{3-20}$$

System 3:

$$2F_2'G_0 + 2F_2G_0' - 2G_2'F_0 - 2G_2F_0' = G_2'' \tag{3-15}$$

$$-F_1'^2 - G_1^2 - 2F_2''F_0 - 2F_2'F_0' + 2F_2F_0'' - 2G_2G_0 = 2h_2 + F_2''' \tag{3-18}$$

$$h_2' = 0 \tag{3-22}$$

The boundary conditions for the above systems are:

System 1:

$$F_0(0) = 0 \quad ; \quad F_0(S) = 0$$

$$F_0'(0) = 0 \quad ; \quad F_0'(S) = 0$$

$$G_0(0) = 1 \quad ; \quad G_0(S) = 0$$

System 2:

$$F_1(0) = 0 \quad ; \quad F_1(S) = 1$$

$$F_1'(0) = 0 \quad ; \quad F_1'(S) = 0$$

$$G_1(0) = 0 \quad ; \quad G_1(S) = 0$$

System 3:

$$F_2(0) = 0 \quad ; \quad F_2(S) = 0$$

$$F_2'(0) = 0 \quad ; \quad F_2'(S) = 0$$

$$G_2(0) = 0 \quad ; \quad G_2(S) = 0$$

This then is a mathematically well posed problem to be solved numerically. The pressure will be determined only within a constant. This is of no importance since pressure difference is the significant quantity. The function $h(z)$ represents the variation of the

pressure in the axial direction.

Numerical Solution Procedure:

System 1, represents the problem of flow between rotating and stationary disks, without leakage and it has been solved by many investigators, e.g. [1,2,3,4,9,10] etc. The solution procedure for system 1 used in the present study has been taken from Cooper [10].

System 1 is a set of ordinary non-linear differential equations. This nonlinearity tends to bring numerical instability to the system. Cooper starts his numerical integration from both the stationary wall and rotating disk sides and matches them in the middle. The matching point is taken at 1/3 gap width from the stationary wall side. It has been observed that the integration process can go smoothly to larger distance from the rotating disk than from the wall. Solutions obtained from both sides are matched for their values and for all the derivatives up to an order less than the order of the equations. These solutions require good guesses at the wall and the disk, especially at larger gap widths. Accurate guesses are generated by a different scheme for gap widths greater than $24\sqrt{\nu/\Omega}$. Here Cooper assumes a core angular velocity and shoots from both sides up to the edges of assumed boundary layers. The guesses on the wall and the guesses are varied to attain this core angular velocity. These solutions are then tested for angular momentum matching. If they do not satisfy this matching condition, then the core angular velocity is perturbed and the same iteration scheme repeated. Once

the guesses at the wall and the disk are known, the previous procedure of matching at a point is followed.

A variable size grid is used. The grid is finer close to the boundaries and becomes coarser as one moves towards the center. Such a grid is essential as solutions are very sensitive to initial guesses and also large velocity gradients exist close to the boundaries. Finer grid means smaller discretization error. The procedure for system 1 is given in detail by Cooper [10].

System 2 and system 3 are sets of linear ordinary differential equations, so superposition can be used. For solving these systems, the Adams-Moulton predictor-corrector method has been chosen. This procedure requires the solutions at first 4 points for starting, which in turn are obtained by the Runge-Kutta method. To start the Runge-Kutta estimation, values of the functions and their derivatives at the initial point are needed. As the solution of one system provides coefficients for the differential equations of the subsequent system, the grid of system 1 has been retained.

For the gap widths smaller than $10\sqrt{\nu/\Omega}$, solutions of system 2 and 3 were obtained by starting the integration from the disk side and matching at the wall. This method is simpler as the number of guesses required at the disk and number of match conditions at the wall are only three. But this procedure fails for larger gap widths due to large accumulated errors. Hence this method was abandoned for another procedure in which integration is started from both sides and the solutions are matched inside the

region. This match point has been chosen on the basis that the solutions from both sides are of same order of magnitude. In this scheme the number of matched conditions is increased to 7 in system 2 and 6 in system 3. The Adams-Moulton along with Runge-Kutta integration methods are used for forward marching from the rotating disk and backward marching from the stationary wall. The values of the functions and their derivatives up to one order less than the equations are required at the wall and the disk, to start the integration. Some of these are provided by the boundary conditions and remainder are guessed.

System 2 is set of linear ordinary homogeneous differential equations with non-homogeneous boundary conditions. This system is solved by directly integrating the equations. Some of the starting conditions, provided by boundary-conditions, are as follows:

$$\left.\begin{aligned} F_1(0) &= 0 \\ F_1'(0) &= 0 \\ G_1(0) &= 0 \end{aligned}\right\} \qquad \text{(3-23)} \quad \text{ON DISK SIDE}$$

$$\left.\begin{aligned} F_1(S) &= 1 \\ F_1'(S) &= 0 \\ G_1(S) &= 0 \end{aligned}\right\} \qquad \text{(3-24)} \quad \text{ON WALL SIDE}$$

The remaining conditions are:

$$F_1''(0) = ? \qquad F_1''(s) = ?$$
$$G_1'(0) = ? \qquad G_1'(s) = ? \qquad (3\text{-}25)$$
$$h_1(0) = \text{Constant} = ? \qquad h_1(s) = \text{Constant} = ?$$

As (3-25) are not known, they are guessed and the solution calculated from both sides up to the match points. Each of these solutions are tentative as the guesses may not be exact. Solutions for different sets of guesses are combined by superposition. These combined solutions from both sides are then matched by equating their values and different derivatives. Specifically, values of F_1, F_1', F_1'', G_1, G_1', h_1 are matched and an additional condition at the wall $F_1(s) = 1$, which arises due to nonhomogeneous boundary condition, is satisfied. There are seven matching conditions. So a set of seven solutions are needed, three from the rotating disk and four from the stationary wall.

If x (i) are the constants used to combine the solutions and $F_1(i, z)$, $G_1(i, z)$, $h_1(i, z)$ represents corresponding set of solutions.

Then:

$$
\begin{aligned}
\sum_{i=1}^{3} X(i)\, F_1(i, Z_1) &= \sum_{i=4}^{7} X(i)\, F_j(i, Z_1) \\
\sum_{i=1}^{3} X(i)\, F_1'(i, Z_1) &= \sum_{i=4}^{7} X(i)\, F_j'(i, Z_1) \\
\sum_{i=1}^{3} X(i)\, F_1''(i, Z_1) &= \sum_{i=4}^{7} X(i)\, F_j''(i, Z_1) \\
\sum_{i=1}^{3} X(i)\, G_1(i, Z_1) &= \sum_{i=4}^{7} X(i)\, G_1(i, Z_1) \\
\sum_{i=1}^{3} X(i)\, G_1'(i, Z_1) &= \sum_{i=4}^{7} X(i)\, G_1'(i, Z) \\
\sum_{i=1}^{3} X(i)\, h_1(i, Z) &= \sum_{i=4}^{7} X(i)\, h_1(i, Z) \\
\sum_{i=4}^{7} X(i)\, F_j(i, S) &= 1
\end{aligned}
\qquad (3\text{-}26)
$$

where Z_1 is the position of match point.

As seven solutions are needed, so 3 sets of guesses on the disk and 4 sets of guesses on the wall for (3-25) are made. The set of algebraic equations (3-26) is solved by the Gauss-Jordan method. Once these constants X(i), i=1,7 are known, the solution of system 2 is obtained by combining the different tentative solutions as follows:

For disk side

$$F_1(z) = \sum_{i=1}^{3} X(i)\, F_1(i,z)$$

$$G_1(z) = \sum_{i=1}^{3} X(i)\, G_1(i,z) \qquad 0 \le z \le z_1$$

$$h_1 = \sum_{i=1}^{3} X(i)\, h_1(i)$$

For wall side

$$F_1(z) = \sum_{i=4}^{7} X(i)\, F_1(i,z)$$

$$G_1(z) = \sum_{i=4}^{7} X(i)\, G_1(i,z) \qquad z_1 \le z \le S$$

For system 3 a similar scheme has been used. As system 3 is a non-homogeneous set of linear ordinary differential equations with homogeneous boundary conditions, in principle it can be solved directly, if enough starting conditions are known. System 3 has been divided into two parts; one with non-homogeneous equations and enough homogeneous conditions to start the integration, and the second part consists of homogeneous equations with matching conditions as forced by the first part. So the linear combination of both the parts will satisfy all the boundary conditions of system 3.

The scheme is as follows:

$$F_2 = F_{2h} + F_{2p}$$

$$G_2 = G_{2h} + G_{2p}$$

$$h_2 = h_{2h} + h_{2p}$$

As h_2 is a constant, so it can be assumed that total contribution to h_2 comes from homogeneous part and $h_{2p} = 0$

The equations for the non-homogeneous part are:

$$\left.\begin{aligned} F_{2p}''' &= (G_1^2 + 2G_0G_{2p}) - 2(F_0''F_{2p} - F_{2p}''F_0) \\ &\quad + 2F_{2p}'F_0' + F_1'^2 \\ G_{2p}'' &= 2F_{2p}'G_0 + 2G_0'F_{2p} - 2F_0G_{2p}' - 2F_0'G_{2p} \end{aligned}\right\} \qquad (3\text{-}27)$$

Boundary conditions for this part are are follows:

$$\left.\begin{aligned} F_{2p}(0) &= F_{2p}'(0) = F_{2p}''(0) = 0 \\ G_{2p}(0) &= G_{2p}'(0) = 0 \end{aligned}\right\} \qquad (3\text{-}28)$$

$$\left.\begin{aligned} F_{2p}(s) &= F_{2p}'(s) = F_{2p}''(s) = 0 \\ G_{2p}(s) &= G_{2p}'(s) = 0 \end{aligned}\right\} \qquad (3\text{-}29)$$

Integration up to the match point from both sides is carried out by the Adams-Moulton method in combination with the Runge-Kutta

procedure. The Runge-Kutta scheme is started with the help of (3-28) and (3-29). Thus the solution of (3-27) is found. The equations for the homogeneous part are:

$$\left.\begin{aligned} F_{2h}''' + 2h_2 &= 2(F_0'' F_{2h} - F_{2h}'' F_0) - 2F_{2h}' F_0 \\ &\quad - 2G_0 G_{2h} \\ G_{2h}'' &= 2F_{2h}' G_0 + 2G_0' F_{2h} - 2F_0 G_{2h}' \\ &\quad - 2F_0' G_{2h} \\ h_2' &= 0 \end{aligned}\right\} \qquad (3\text{-}30)$$

The boundary conditions for homogeneous part:

$$\left.\begin{aligned} F_{2h}(0) &= F_{2h}'(0) = 0 \\ G_{2h}(0) &= 0 \end{aligned}\right\} \qquad (3\text{-}31)$$

$$\left.\begin{aligned} F_{2h}(S) &= F_{2h}'(S) = 0 \\ G_{2h}(S) &= 0 \end{aligned}\right\} \qquad (3\text{-}32)$$

As the boundary conditions are not sufficient to start the integration, the following additional conditions are guessed:

$$\left.\begin{aligned} &F_{2h}''(0) = ? \qquad G_{2h}'(0) = ? \qquad h_2 = ? \\ &F_{2h}''(S) = ? \qquad G_{2h}'(S) = ? \qquad h_2 = ? \end{aligned}\right\} \qquad (3\text{-}33)$$

Since the three guesses on the disk and wall sides are arbitrary, the solution obtained is only tentative. Hence three sets of guesses on the disk side and another three sets on the wall side are made. Solutions of system 3 in the disk field and in the wall field are obtained by taking a linear combination of these three solutions and adding the particular solution to it. The solutions in both the fields should match at the matching point. Quantities which are matched are:

$F_2, F_2', F_2'', G_2, G_2', h_2$

The details are as follows:

If x(i) represents the constants, used for combining different solutions, and F_{2h} (i,z), G_{2h} (i,h), h_2 (i) the solution functions in both the fields, then the solutions are:

In disk field:

$$\left.\begin{aligned} F_{2h}(z) &= \sum_{i=1}^{3} X(i)\, F_{2h}(i,z) \\ G_{2h}(z) &= \sum_{i=1}^{3} X(i)\, G_{2h}(i,z) \\ h_2 &= \sum_{i=1}^{3} X(i)\, h_2(i) \end{aligned}\right\} \quad 0 \leq z \leq z_1 \qquad (3\text{-}34)$$

In wall field:

$$\left.\begin{aligned} F_{2h}(z) &= \sum_{i=4}^{6} X(i)\, F_{2h}(i,z) \\ G_{2h}(z) &= \sum_{i=4}^{6} X(i)\, G_{2h}(i,z) \\ h_2 &= \sum_{i=4}^{6} X(i)\, h_2(i) \end{aligned}\right\} \quad z_1 \leq z \leq S \qquad (3\text{-}35)$$

Solutions of system 3 are matched by following conditions.

Matching conditions:

$$\begin{aligned}
&\sum_{i=1}^{3} X(i)\, F_{2h}(i,Z_1) - \sum_{i=4}^{6} X(i)\, F_{2h}(i,Z_1) = F_{2PW}(Z_1) - F_{2PD}(Z_1) \\
&\sum_{i=1}^{3} X(i)\, F'_{2h}(i,Z_1) - \sum_{i=4}^{6} X(i)\, F'_{2h}(i,Z_1) = F'_{2PW}(Z_1) - F'_{2PD}(Z_1) \\
&\sum_{i=1}^{3} X(i)\, F''_{2h}(i,Z_1) - \sum_{i=4}^{6} X(i)\, F''_{2h}(i,Z_1) = F''_{2PW}(Z_1) - F''_{2PD}(Z_1) \\
&\sum_{i=1}^{3} X(i)\, G'_{2h}(i,Z_1) - \sum_{i=4}^{6} X(i)\, G'_{2h}(i,Z_1) = G'_{2PW}(Z_1) - G'_{2PD}(Z_1) \\
&\sum_{i=1}^{3} X(i)\, G_{2h}(i,Z_1) - \sum_{i=4}^{6} X(i)\, G_{2h}(i,Z_1) = G_{2PW}(Z_1) - G_{2PD}(Z_1) \\
&\sum_{i=1}^{3} X(i)\, h_2(i) - \sum_{i=4}^{6} X(i)\, h_2(i) = 0
\end{aligned} \tag{3-36}$$

These are set of 6 algebraic equations in 6 unknowns and are solved by Gauss-Jordan method. The resulting final solution of system 3 is as follows:

In disk field

$$\left.\begin{aligned} F_2(Z) &= F_{2pD}(Z) + \sum_{i=1}^{3} X(i)\, F_{2h}(i,Z) \\ G_2(Z) &= G_{2pD}(Z) + \sum_{i=1}^{3} X(i)\, G_{2h}(i,Z) \\ h_2 &= \sum_{i=1}^{3} X(i)\, h_2(i) \end{aligned}\right\} \quad 0 \leq Z \leq Z_1$$

In wall field

$$\left.\begin{aligned} F_2(Z) &= F_{2pW}(Z) + \sum_{i=4}^{6} X(i)\, F_{2h}(i,Z) \\ G_2(Z) &= G_{2pW}(Z) + \sum_{i=4}^{6} X(i)\, G_{2h}(i,Z) \end{aligned}\right\} \quad Z_1 \leq Z \leq S$$

where z_1 is the position of the matching point.

Solution of this system is generally an order of magnitude lower then lower order solution, but at certain points they do become comparable. Still there is felt to be no need to go to higher order solutions. Once the solutions of all the three systems are known, velocity and pressure distributions between disk and the wall can be found. The remaining function in the expression for pressure is computed as follows:

$$h(z) = -2F_0' - 2F_0^2 + \text{Const} \tag{3-21}$$

The constant cannot be evaluated unless the pressure is known at a point inside the pump. One can neglect this constant in finding pressure distribution, as only the pressure difference is really

significant. The velocity and the pressure field are as follows:

$$\left.\begin{aligned} U(r,Z;Q) &= r F_0' + (Q/r) F_1' + (Q^2/r^3) F_2' \\ V(r,Z;Q) &= r G_0 + (Q/r) G_1 + (Q^2/r^3) G_2 \\ W(r,Z;Q) &= -2F_0 + (2Q^2/r^4) F_2 \end{aligned}\right\} \qquad (3\text{-}37)$$

$$P(r,Z;Q) = r^2 h_0 + Q h_1 \ln r + (Q^2/r^3) h_2 + h \qquad (3\text{-}38)$$

where h_0, h_1, h_2 are all constants and F_o, F_1, F_2, G_o, G_1, G_2, are independent of leakage. Pressure variations for the no inflow case have been provided by Cooper [10] in terms of β, the core angular velocity ratio

$$\frac{\partial P}{\partial r} = -\beta^2 r = 2r h_0$$

$$h_0 = -\frac{\beta^2}{2}$$

where

$$\beta = \frac{\Omega_{core}}{\Omega_{disk}}$$

Hence the pressure and velocity fields are known.

A particular case of this problem has been solved by Peube & Kreith [7]. Their no-inflow case consists of two disks and the enclosed fluid rotating at same angular velocity;the effect of

inflow is studied by the same perturbation scheme as used in this study. As a check of the present numerical procedure, the Peube and Kreith problem was solved as well.

The solutions obtained for system 2 and system 3 match up to 5 significant figures with the analytical solution obtained by Peube & Kreith [7].

IV. AN INTEGRAL OF THE ENERGY EQUATION

An attempt has been made to find the flow situation where Reynolds analogy can be used to find the temperature field. Equations (2-1) - (2-5) are nondimensionalised with different reference quantities so that the largest significant terms of the equation could be retained.

$$\bar{Z}_{ref} = \bar{S} \qquad \bar{r}_{ref} = \bar{R}$$

Reference velocities; U_{ref}, V_{ref}, W_{ref}

From continuity:

$$\frac{\partial(ur)}{\partial r}\left(U_{ref}\right) + \frac{\partial(wr)}{\partial Z}\left(W_{ref}\frac{\bar{R}}{\bar{S}}\right) = 0$$

which implies,

$$\frac{U_{ref}}{\bar{R}} = W_{ref}\frac{\bar{R}}{\bar{S}}$$

Momentum

$$\left(u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial Z}\right) - \frac{v^2}{r}\left(\frac{V_{ref}}{U_{ref}}\right)^2 = -\frac{1}{\rho U_{ref}^2}\frac{\partial \bar{P}}{\partial r} + \left(\frac{\nu}{\bar{R}U_{ref}}\right)\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\left(\frac{\partial ur}{\partial r}\right)\right) + \frac{\partial^2 u}{\partial Z^2}\left(\frac{\bar{R}}{\bar{S}}\right)^2\right]$$

$$\left(u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial Z}\right) + \frac{uv}{r} = \left(\frac{\nu}{\bar{R}U_{ref}}\right)\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial(vr)}{\partial r}\right) + \frac{\partial^2 v}{\partial Z^2}\left(\frac{\bar{R}}{\bar{S}}\right)^2\right]$$

$$\left(u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\left(\frac{\bar{R}}{\bar{S}}\right)\frac{1}{\rho U_{ref} W_{ref}}\frac{\partial \bar{P}}{\partial z} + \left(\frac{\nu}{\bar{R}U_{ref}}\right)\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\left(\frac{\bar{R}}{\bar{S}}\right)^2\right]$$

If

$$\frac{\bar{R}}{\bar{S}} \gg 1$$

$$\left(\frac{\bar{R}}{\bar{S}}\right)\left(\frac{\nu}{U_{ref}\bar{S}}\right) = O(1)$$

<u>**Energy**</u>

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{\nu U_{ref}}{CT_{ref}\bar{R}}\left[u\frac{\partial P}{\partial r} + w\frac{\partial P}{\partial z}\right] + \left(\frac{K}{\rho C}\right)\left(\frac{1}{\bar{R}U_{ref}}\right)\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \left(\frac{\bar{R}}{\bar{S}}\right)^2\frac{\partial^2 T}{\partial z^2}\right] + \frac{\nu U_{ref}}{CT_{ref}\bar{R}}\left[2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{u}{r}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\left(\frac{V_{ref}}{U_{ref}}\right)^2\left(\frac{\bar{R}}{\bar{S}}\right)^2 + \left\{\frac{\partial u}{\partial z}\left(\frac{\bar{R}}{\bar{S}}\right) + \frac{\partial w}{\partial r}\left(\frac{\bar{S}}{\bar{R}}\right)\right\}^2 + \left\{\frac{\partial v}{\partial r}\left(\frac{V_{ref}}{U_{ref}}\right)^2 - \frac{v}{r}\left(\frac{V_{ref}}{U_{ref}}\right)^2\right\}^2\right]$$

$$\frac{K}{\rho C\bar{R}U_{ref}} = \frac{1}{P_r}\times\frac{1}{Re}$$

$$\frac{\nu\, U_{ref}}{C\, T_{ref}\, \bar{R}} = \frac{E_c}{Re}$$

The above set of equations can be simplified for

$P_r = 1$, and $\bar{R}/\bar{S} \gg 1$

The resulting dimensional equation with bar removed are

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \nu\left(\frac{\partial^2 u}{\partial z^2}\right) \qquad (4\text{-}1)$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \nu\frac{\partial^2 v}{\partial z^2} \qquad (4\text{-}2)$$

$$\frac{\partial P}{\partial z} = O\left(\bar{S}/\bar{R}\right), \quad \frac{\partial P}{\partial r} = O(1)$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{1}{\rho c}\left(u\frac{\partial P}{\partial r} + w\frac{\partial P}{\partial z}\right) \qquad (4\text{-}3)$$
$$+ \frac{\nu}{c}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \frac{K}{\rho c}\left(\frac{\partial^2 T}{\partial z^2}\right)$$

These are boundary layer equations. Although equations (4-1), (4-2) give same solution as obtained from exact Navier-Stokes equations for Von-Karman's similarity, the solution of the energy equation (4-3) is limited to boundary layer region and only valid away from the axis.

Equations 4-1, 4-2, 4-3 are nondimensionalised by usual reference quantities (as in 2-6 to 2-9).

So nondimensional equations are:

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial P}{\partial r} + \frac{\partial^2 u}{\partial z^2}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = \frac{\partial^2 v}{\partial z^2}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial z^2} + \frac{E_c}{Re}\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \frac{E_c}{Re}\left[u\frac{\partial P}{\partial r} + w\frac{\partial P}{\partial z}\right]$$

Where

$$\frac{E_c}{Re} = \frac{\nu\Omega}{C\,T_{ref}}$$

Now these equations are combined to eliminate pressure terms along with the boundary layer assumption that:

$$u\frac{\partial P}{\partial r} \gg w\frac{\partial P}{\partial z}$$

So combined equation is:

$$u\frac{\partial}{\partial r}\left[T + \frac{u^2+v^2}{2}\left(\frac{E_c}{Re}\right)\right] + w\frac{\partial}{\partial z}\left[T + \frac{u^2+v^2}{2}\left(\frac{E_c}{Re}\right)\right] = \frac{\partial^2}{\partial z^2}\left[T + \frac{u^2+v^2}{2}\left(\frac{E_c}{Re}\right)\right] \qquad (4\text{-}4)$$

Defining a new variable, nondimensional total enthalpy as:

$$H = T\left(\frac{Re}{E_c}\right) + \frac{U^2+V^2}{2} \tag{4-5}$$

This H is substituted into (4-4) and new equation obtained:

$$u\frac{\partial H}{\partial r} + w\frac{\partial H}{\partial z} = \frac{\partial^2 H}{\partial z^2} \tag{4-6}$$

Boundary conditions for above equation depend upon the physical situation under consideration. Two cases solved in this study are, insulated disk with the wall cooled to some constant temperature, and other with the disk and the wall cooled to same temperature.

In order to solve (4-6), it is expanded in terms of a series using the expansions for u & v.

From (3-37):

$$U^2 = r^2 F_0'^2 + 2Q F_0' F_1' + (Q^2/r^2)(F_1'^2 + 2F_0' F_2') + \cdots$$

$$V^2 = r^2 G_0^2 + 2Q G_0 G_1 + (Q^2/r^2)(G_1^2 + 2G_0 G_2) + \cdots$$

Hence H is potentially of the form:

$$H = Const + r^2 H_0 + 2Q H_1 + (Q^2/r^2) H_2 + \cdots \tag{4-7}$$

Now expansions of u, V & ω are substituted into (4-6) and equations corresponding to different power of 'r' are obtained as follows:

(r^2);

$$2H_0F_0' - 2F_0H_0' = H_0'' \qquad (4\text{-}8)$$

(r^0);

$$2H_0F_1' - 2H_1'F_0 = H_1'' \qquad (4\text{-}9)$$

(r^{-2});

$$F_2'H_0 + F_2H_0' - H_2F_0' - F_0H_2' = \tfrac{1}{2}H_2'' \qquad (4\text{-}10)$$

Boundary conditions for different order equations are obtained for both cases. For the first case of insulated disk, boundary conditions are as follows:

$$\frac{\partial T}{\partial Z}(0, r) = 0$$

or

$$\left.\begin{aligned} \frac{\partial H}{\partial Z}(0, r) &= v\frac{\partial v}{\partial Z}(0, r) \\ H(S, r) &= \left(\frac{Re}{E_c}\right) T_W \end{aligned}\right\} \qquad (4\text{-}11)$$

If $\bar{T}_{ref} = \bar{T}_W$, then $T_W = 1$

Boundary conditions for different order equations (4-7, 4-8, 4-9) are obtained from (4-11) and (4-7).

From (4-7) & (4-11)

$$\frac{\partial H}{\partial z}(0,r) = r^2 H_0'(0) + Q H_1'(0) + \frac{Q^2}{r^2} H_2'(0)$$

$$= G_0(0)\left[r^2 G_0'(0) + Q G_1'(0) + \frac{Q^2}{r^2} G_2'(0)\right]$$

or

$$H_0'(0) = G_0'(0) \tag{4-8-1}$$

$$H_1'(0) = G_1'(0) \tag{4-9-1}$$

$$H_2'(0) = G_2'(0) \tag{4-10-1}$$

Also at z=s:

$$H(s,r) = \frac{R_e}{E_\epsilon} = \text{Const} + r^2 H_0(s) + Q H_1(s) + \frac{Q^2}{r^2} H_2(s)$$

or

$$\text{Const} = \frac{R_e}{E_c}$$

$$H_0(s) = 0 = G_0(s) \tag{4-8-2}$$

$$H_1(s) = 0 = G_1(s) \tag{4-9-2}$$

$$H_2(s) = 0 = G_2(s) \tag{4-10-2}$$

Equations (4-8), (4-9), (4-10) and their boundary conditions are compared with 3-13, 3-14, 3-15 and it is found that former equations can be obtained by replacing G_i by corresponding H_i. Hence:

$$H_0 = G_0\,; \quad H_1 = G_1\,; \quad H_2 = G_2$$

And so the solution of equation (4-6) is:

$$H = \frac{Re}{E_c} + \pi^2 G_0 + Q G_1 + \frac{Q^2}{\pi^2} G_2 \tag{4-11}$$

Temperature distributions can be obtained from (4-11), using (4-5):

$$T = 1 + \frac{E_c}{Re}\left(\pi^2 G_0 + Q G_1 + \frac{Q^2}{\pi^2} G_0\right) - \frac{E_c}{2Re}\Big[\pi^2 F_0'^2 + 2Q F_0' F_1' + \frac{Q^2}{\pi^2}\left(F_1'^2 + 2 F_0'^2 F_2'\right) + \pi^2 G_0^2 + 2Q G_0 G_1 + \frac{Q^2}{\pi^2}\left(G_1^2 + 2 G_0^2 G_2\right)\Big]$$

or

$$\frac{T}{T_w} = 1 + \frac{E_c}{Re}\Big[\left(\pi^2 G_0 - \frac{\pi^2 F_0'^2}{2} + \pi^2 G_0^2\right) + Q\left(G_1 - F_0' F_1' - G_0 G_1\right) + \frac{Q^2}{\pi^2}\left(G_2 - \frac{F_1'^2}{2} - F_0'^2 F_2' - \frac{G_1^2}{2} - G_0^2 G_2\right)\Big] \tag{4-12}$$

So effect of leakage on temperature distribution is known.

For the second case, when the disk and the wall are cooled to the same temperature, the boundary conditions are different but the solution is obtained in the same way.

The boundary conditions are:

$$T(0) = \bar{T}(0)/T_w = 1 \tag{4-13}$$

or

$$H(0,r) = \text{Const} + r^2 H_0(0) + Q\, H_1(0) + \frac{Q^2}{r^2} H_2(0)$$

$$= \frac{Re}{E_c} + \frac{1}{2} r^2 G_0^2(0)$$

$$= \frac{Re}{E_c} + \frac{1}{2} r^2 G_0(0)$$

or

$$\text{Const} = \frac{Re}{E_c}$$

$$H_0(0) = \tfrac{1}{2} G_0(0) \qquad \text{4-8-3}$$

$$H_1(0) = 0 \qquad \text{4-9-3}$$

$$H_2(0) = 0 \qquad \text{4-10-3}$$

And at z=s, they are the same as (4-8-2), (4-9-2), (4-10-2)

$$H_0(s) = 0 = \tfrac{1}{2} G_0(s)$$

$$H_1(s) = 0 = G_1(s)$$

$$H_2(s) = 0 = G_2(s)$$

So H_0, H_1 & H_2 are equivalent to 1/2 G_o, G_1, G_2 respectively. Hence the solution of second case is:

$$H = \frac{R_e}{E_c} + \frac{1}{2}\Re^2 G_o + Q G_1 + \frac{Q^2}{\Re^2} G_2 \tag{4-14}$$

and the corresponding temperature distribution as obtained from (4-14) is:

$$\begin{aligned}\frac{\bar{T}}{\bar{T}_W} = 1 &+ \frac{E_c}{R_e}\left(\frac{1}{2}\Re^2 G_o + Q G_1 + \frac{Q^2}{\Re^2} G_2\right) \\ &- \frac{E_c}{R_e}\Big[\Re^2 F_o'^2 + 2Q F_o' F_1' + \frac{Q^2}{\Re^2}\big(F_1'^2 \\ &+ 2 F_o'^2 F_2'\big) + \Re^2 G_o^2 + 2Q G_o G_1 \\ &+ \frac{Q^2}{\Re^2}\big(G_1^2 + 2 G_o^2 G_2\big)\Big]\end{aligned} \tag{4-15}$$

Once the temperature distribution for a given case is known, then the disk temperature and heat transfer rate at the wall in case 1 and heat transfer rates at the disk and at the wall in case 2 can be found.

For Case 1:

The temperature at the disk is:

$$\frac{\bar{T}_D}{\bar{T}_W} = 1 + \frac{1}{2}\,\frac{E_c}{R_e}\left(\Re^2\right) \tag{4-16}$$

and the heat transfer rate at the wall is:

$$\left.\frac{\partial T}{\partial Z}\right|_{Z=\delta} = \frac{E_c}{R_e}\left(\Re^2 G_o'(s) + Q G_1'(s) + \frac{Q^2}{\Re^2} G_2'(s)\right) \tag{4-17}$$

For Case 2:

The heat transfer rate at the wall is:

$$\left.\frac{\partial T}{\partial z}\right|_{z=s} = \frac{E_c}{Re}\left(\frac{1}{2}\mathcal{R}^2 G_0'(s) + Q\, G_1'(s) + \frac{Q^2}{\mathcal{R}^2} G_2'(s)\right) \tag{4-18}$$

and at the disk:

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = -\frac{E_c}{Re}\left(\frac{1}{2}\mathcal{R}^2 G_0'(0)\right) \tag{4-19}$$

From the preceding results, it can be concluded that leakage affect the heat transfer rate at the wall. This rate is reduced for second case as some heat is conducted at the disk now. But the surprising result is that for solutions within the framework of the separation of variables chosen, the temperature of the disk in first case and heat transfer rate to the disk in second case are independent of leakage rate. This can be explained as follows: Effect of leakage is to reduce the shear stresses at the disk, which means smaller dissipation. Also, leakage reduces radial velocity and so convection rate. Hence net effect is that smaller amount of the reduced dissipated heat is convected. The disk gets the same amount of heat irrespective of leakage for Prandtl number one.

V. RESULTS AND DISCUSSION

The system of equations (3-13) to (3-22) along with their boundary conditions have been solved for three values of S, where S^2 is Taylor number. These solutions are tabulated in Tables 1 to 9. The system-1 functions are shown in Figures 2 to 4, the system-2 functions in Figures 5 to 7 and the system-3 functions in Figures 8 to 10. The system-3 solution for S = 30 is not available due to numerical instability.

Convergence of the integration procedure is strongly dependent on the guesses which are made to start the integration for each system of differential equations. This is particularly true of system-1, which consists of non-linear differential equations. As superposition of solutions is not possible for system-1, convergence can be obtained only when the guesses are fairly close to the final solution. As each system depends upon the solution to the previous system, the net effect of accumulated error shows up most strongly in the third system. It appears that for S = 30, the accumulated discretization error for system-3 precludes a proper solution. This problem can be avoided by using finer grid, which will require a computer with larger core.

The result of different systems will be discussed with respect to flow field, shear stresses and pressure distribution.

Flow Field

The flow field between the disk and the wall has been analyzed for three different non-dimensional widths ($S = 1.0, 17.15, 30.0$). Here S^2 or Taylor number represents the ratio between inertia forces to viscous forces. For $S<24$, the basic flow is of the merged boundary layer type [12] Thus, as seen in the velocity profiles (Figures 2 to 4), for $S = 1, 17.15$ the boundary layers on the disk and the wall are merged, while for $S = 30$, separate boundary layers with a core in between are evident. For very small gap widths, $S \leq 1$, viscous effects dominate, and so the azimuthal velocity profile is nearly a straight line (Figure 4) while the axial and radial velocities are quite small (Figures 2 and 3 respectively). For large gap width the effects of inertia become important and this is reflected in the appearance of secondary flows.

For large S, viscous influences decrease as the distance from the wall or the disk increases. Away from the wall or the disk, the pressure force is balanced by the centrifugal force, but close to the stationary wall, the centrifugal force is unable to balance the pressure gradient and there is a radially inward flow. Similarly excess centrifugal force near the disk creates an outward radial flow, as evident from Figure 3. In order to explain the other peaks and troughs, the relative importance of the different inertia terms has to be considered. For $S \leq 1$, viscous terms dominate so no secondary ripples in the profiles are seen. But for $S \geq 1$, there are regions of inflow and outflow between the disk and the wall, due to strong inertia effects.

After the first trough in radial velocity near the wall, the viscous effects are weak and radial convection of angular momentum dominates over axial convection as axial velocities are small, resulting in faster rotation of fluid particles, hence a peak in the azimuthal velocity profile. This higher angular velocity increases the centrifugal force and so reduces the inward radial velocity. After this there is a region of radial outflow. In order to satisfy continuity, there is a reduction in axial velocity to a local minimum. In this region, since the radial convection is from a region of lower angular momentum, the angular velocity decreases and there is a trough in azimuthal velocity profile. In turn, because of the lower angular velocity, centrifugal forces are weak, compared to the pressure gradient resulting again in radial inflow. As the momentum equations or momentum balance are applicable at a point, the regions of inflow and outflow can only be qualitatively explained in terms of dominant terms. Also since the influences of three factors, radial and axial convection, and centrifugal force, are involved so there is overlapping of regions of peak and trough of one profile with another.

This situation is modified when there is inflow. This inflow comes from larger radius, hence with excess angular momentum, which is imparted to the fluid around it. This results in increased angular velocity as is evident from Figure 7, which shows that the second order function G_1 is always negative, and for inflow, Q is also negative. It is not easy to explain the effect of inflow on radial velocity (Figure 6). There are two effects, one is due to the faster rotation of fluid which accelerates the fluid outward and the second is the negative radial

momentum carried by inflow, which tends to suppress radial outflow near the disk and augment radial inflow near the wall. This negative radial momentum effect dominates the flow close to the disk and especially near the wall as the fluid is rotating slower in this region. In the center region, it seems that the effect due to faster rotation dominates. For the case of S = 1, viscous influences are strong and the effect of inflow is equivalent to superposing an inward channel flow between two stationary disks (Figure 6) on the basic flow. Axial velocity is not affected by the inflow up to third order.

This effect of inflow is very strong close to the axis, where the basic angular momentum is low and all the fluid is moving inward as is evident from figures 11 to 13. Stream lines in these figures indicate that fluid recirculates beyond a certain radius where centrifugal force becomes significant. The radius beyond which recirculation starts, is the radius where radial shear stress first changes sign. This expression can be computed from the expression for radial velocity.

$$u = rF_0' + QF_1'/r + Q^2F_2'/r^3 + ----$$

For the case of small Q, Q^2F_2'/r^3 can be neglected compared to QF_1'/r leading to the following estimate of the radius outside of which, there is flow recirculation.

$$\text{Radius of recirculation} = \sqrt{-\frac{F_1'Q}{F_0'}} \qquad (5\text{-}1)$$

Despite the inaccuracies of Soo's analysis [5], it provides fairly good estimate of recirculation radius r, where

$$r^2 = \frac{60Q}{S^3}$$

For the case of disk and wall, recirculation starts near the disk and estimates of recirculation radius can be made by evaluating (5-1) at the disk. As Soo's 5 analysis is for small gap widths, the radius of recirculation is computed for S = 1, and it is found to be fairly close to Soo's estimate.

Radius of recirculation (S = 1) = $\sqrt{59.7469\,Q}$

As is to be expected, it is found that recirculation radius increases with Q and decreases with gap width (S). Streamlines are computed as lines of constant stream function, using relation

$$\psi = r^2 F_0 + Q F_1 + (Q^2/r^2) F_2 + \cdots$$

For small Q, the radial location of a streamline at different axial positions is approximated (except near r = 0) by

$$r^2 = \frac{\psi - Q F_1}{F_0} \qquad (5\text{-}3)$$

These are not the actual streamlines but the "projection" of the streamlines in a meridional plane (θ = const)

These are not the actual streamlines by the projection of the stream-lines by the projection of the streamlines in a meriodional plane. (θ = const). Radius of recirculation can also be estimated by locating position of stagnation streamline on the disk.

Radius of recirculation = $\lim\limits_{\substack{z \to 0 \\ \psi = 0}} \dfrac{\psi - Q F_1}{F_0} = - \dfrac{Q F_1''}{F_0''}$

Shear Stress Distribution

There is a significant influence of inflow on shear stresses at the wall and the disk. Since the fluid rotates faster due to inflow, azimuthal shear stress is reduced at the rotating disk and increased at the stationary wall. Similar effects are observed for the radial shear stresses due to radial momentum carried by the inflow. This is evident from the tables 10 and 11, which indicate the relative magnitudes of the contributions from the different order solutions. Also to be noted is that the influence of inflow on shear stresses is more pronounced for larger gap widths. This can be explained in terms of the decreasing effect of viscous forces on the solution. Since the excess angular momentum of the inflow is not diffused well in the middle region, the effect of rotation of the inflow dominates over the negative radial momentum, resulting in second order outflow. In order to satisfy "continuity", large inflow near the wall and smaller outflow near the disk are observed (Figures 3 and 6). For very small gap width, the effect of inflow is in one direction only, as evident from Figures 6 and 7. These results support Jimbo's [2] conclusion, that the power loss due to skin friction decreases due to inflow since shear stresses decrease at the rotating disk.

Pressure Distribution

The effect of inflow is to increase the radial pressure gradient. The pressure at the axis or periphery is usually known, so the absolute pressure can be obtained from the expression for pressure gradient on the disk.

$$\frac{dP}{dr} = \beta^2 r + \frac{Q H_1}{r} - \frac{2Q^2 H_2}{r^3} \qquad \text{ON DISK}$$

This increase in pressure gradient can be explained in terms of the rotation effect of inflow. The fluid tends to rotate faster due to inflow and so generates larger pressure gradient, which is evident from table 13,14 and from the above expression.

Remarks

This study is based on an asymptotic expansion for the different functions and only terms up to 3rd order are retained. This truncation subjects the analysis to a limitation on the amount of inflow that can be considered which depends on the region of application (radial distance) and the accuracy desired. Each term should be smaller than the previous one and the ratio of succeeding terms will be indicative of the accuracy of the estimate of various flow quantities. As an example, the case of S = 17.15 is considered and the dimensionless minimum radius of the region is 20.

The requirement is

$$\frac{Q G_1'/r}{r G_0'} \ll 1$$

If the allowed ratio is 0.1 at r = 20, then

$$\frac{Q G_1'/20}{20 G_0'} = .1$$

and

$$Q = 80$$

The limit on Q is very sensitive to the minimum radius at which computations are desired.

The influence of leakage can be deduced in terms of flow parameters

$$Q = \frac{\bar{Q}}{2\pi\nu\sqrt{\nu/\Omega}}$$

A larger Q means, either large inflow rate or smaller viscosity or large disk angular velocity. In any of these situations the overall flow will be substantially influenced by the inflow.

According to Daily & Nece [11], laminar flow in the non-inflow case exists up to r = 500, after which transition occurs. This transition radius will probably be reduced for the inflow case, since the incoming fluid comes from a turbulent region and brings turbulent fluctuations with it. The region of applicability of the presented laminar solution is between an inner radius which depends upon the accuracy desired for a given Q and an outer radius determined by the onset of transition.

VI. DESIGN APPLICATIONS

This analysis can be used to calculate the axial thrust on a pump impeller due to the asymmetric pressure distribution on both sides as influenced by leakage, and to estimate the power lost due to the skin friction on the impeller. These quantities are of basic importance in evaluating the effect of leakage on pump performance.

Axial Thrust:

This thrust will arise either due to different gap widths between impeller and housing or different leakage rates in the two gaps. The pressure distribution on the impeller shroud is given by

$$P = \frac{\beta^2 r^2}{2} + Q H_1 \ln r + \frac{H_2 Q^2}{r^2} + \text{constant} \tag{6-1}$$

This constant can be evaluated by the knowledge of the pressure at some point inside the pump. If Q and S are different at both sides of the impeller shroud and if P_o represents the pressure at impeller exit at radius R_o, then

$$P = \frac{\beta^2}{2}\left(r^2 - R_o^2\right) + Q H_1 \ln\left(r/R_o\right) + H_2 Q^2\left(\frac{1}{r^2} - \frac{1}{R_o^2}\right) + P_o \tag{6-2}$$

The contribution to axial force due to the flow between the impeller shroud and the housing

$$F = \int_{r_o}^{R_o} 2\pi r p \, dr$$

where r_o is the inner radius of the shroud. Integration yields

$$F = 2\pi P_o\left(\frac{R_o^2 - r_o^2}{2}\right) + \frac{\pi\beta^2}{4}\left(R_o^4 - r_o^4\right) + \pi Q H_1\left[\frac{R_o^2}{2}\right.$$

$$\left.(2\ln R_o - 1) - \frac{r_o^2}{2}(2\ln r_o - 1)\right] + 2\pi H_2 Q^2 \ln\left(\frac{R_o}{r_o}\right)$$

$$- 2\pi\left(\frac{R_o^2 - r_o^2}{2}\right)\left(\frac{\beta^2 R_o^2}{2} + Q H_1 \ln R_o + \frac{H_2 Q^2}{R_o^2}\right) \qquad (6\text{-}3)$$

The net thrust on bearing due to both sides of the impeller is given by

$$\text{Thrust} = F_L - F_R = 2\pi P_o\left(\frac{r_{oR}^2 - r_{oL}^2}{2}\right) - \frac{\pi R_o^4}{4}\left(\beta_L^2 - \beta_R^2\right)$$

$$- \frac{\pi}{4}\left(\beta_L^2 r_{oL}^4 - \beta_R^2 r_{oR}^4\right) - \frac{\pi R_o^2}{2}\left(Q_L H_{1L} - Q_R H_{1R}\right) - \pi\Big[$$

$$Q_L H_{1L}\frac{r_{oL}^2}{2}(2\ln r_o - 1) - Q_R H_{1R}\frac{r_{oR}^2}{2}(2\ln r_{oR} - 1)\Big]$$

$$+ 2\pi H_{2L} Q_L \ln\left(\frac{R_o}{r_{oL}}\right) - 2\pi H_{2R} Q_R^2 \ln\left(\frac{R_o}{r_{oR}}\right) -$$

$$\pi\left(H_{2L} Q_L^2 - H_{2R} Q_R^2\right) + \pi\Big[R_o^2\left(r_{oL}^2 \beta_L^2 - r_{oR}^2 \beta_R^2\right) +$$

$$\ln R_o\left(Q_L H_{1L} r_{oL}^2 - Q_R H_{1R} r_{oR}^2\right) + \pi\Big[R_o^2\left(r_{oL}^2 \beta_L^2 - r_{oR}^2 \beta_R^2\right)$$

$$+ \ln R_o\left(Q_L H_{1L} r_{oL}^2 - Q_R H_{1R} r_{oR}^2\right) + \frac{1}{R_o^2}\left(H_{2L} Q_L^2 r_{oL}^2\right. \qquad (6\text{-}4)$$

$$\left.- H_{2R} Q_R^2 r_{oR}^2\right)\Big]$$

In this expression it is assumed that the outer radius of the impeller and the pressure at that point on both sides of impeller are the same. Non-dimensional core angular velocity β is dependent on the dimensionless gap width but for $S \geqslant 14$ it becomes essentially constant (within 5% variation). This expression can be modified for other physical situations.

Power Loss:

With the knowledge of the azimuthal shear stress distribution on the disk (impeller), power loss due to friction can be evaluated. Thus,

$$\text{Power loss} = PL = \int_{r_o}^{R_o} 2\pi r \, (\tau_{z\theta}) \, v \, dr$$

where

$$v = r G_0 + \frac{Q}{r} G_1 + \frac{Q^2}{r^3} G_2 + \cdots$$

and

$$\tau_{z\theta} = \mu \left(r G_0' + \frac{Q}{r} G_1' + \frac{Q^2}{r^3} G_2' + \cdots \right)$$

So the expression for power loss on one side is

$$PL = 2\pi\mu \Big[G_0 G_0' \frac{r^4}{4} + Q \left(G_1 G_0' + G_0 G_1' \right) \frac{r^2}{2} + Q^2 \left(G_2' G_0 + G_1 G_1' + G_0' G_2 \right) \ln r + \cdots \Big]_{r_o}^{R_o} \qquad (6\text{-}5)$$

The quantity PL can be evaluated for both sides of the impeller separately and the sum will represent the power loss due to skin friction of the impeller.

As the analysis in this study is independent of the sign of Q, it applies as well to disk pumps, and the power required for given amount of flow rate can be evaluated from (6-5). Furthermore, the pressure

gradient and hence the pressure rise can be obtained from (6-1).

Illustrative Problem:

The present analysis will be used to compute the leakage rate and the axial thrust caused by the leakage for a typical centrifugal pump used for lubricating oil.

The physical data for the pump are as follows:

Disk angular velocity	= 1000 rad/sec (955 rpm)
Outer radius (R_o)	= 0.2 ft.
Inner radius (r_o)	= 0.08 ft.
Oil viscosity	= 2.18×10^{-3} lbf.sec/ft^2
Kinematic viscosity	= 10^{-3} ft^2/sec.
Gap width	= 0.01715 ft.
Clearance at seal	= 0.0002 ft.
Pressure rise in impeller	= 200 psi.
Total flow rate	= 50 gpm

The reference quantities for this analysis are:

$$L_{ref} = \sqrt{\nu/\Omega} = 10^{-3}\ ft$$

$$Q_{ref} = 2\pi\nu\sqrt{\nu/\Omega} = 6.3\times10^{-6}\ ft^3/sec$$

$$P_{ref} = \mu\Omega = 2.18\ lbf/ft^2$$

$$Thrust_{ref} = \mu\nu = 2.18\times10^{-6}\ lbf$$

$$H_1 = -.3459$$

$$H_2 = -.1096$$

$$\beta = .314$$

Non-dimensional quantities are:

R_o = 200

r_o = 80

$(\Delta P)_{Imp}$ = 13,200

Dimensionless gap width between impeller shroud and housing

(S) = 17.15

Clearance (δ) = .2

The flow rate across the seal (non-dimensional) is

$$Q = C \times r_o \times \delta \times \sqrt{2 \Delta P_s}$$

where flow coefficient $C = 0.4$

so that

$$\Delta P_s = \frac{Q^2}{81.9} = .0122\, Q^2$$

The pressure difference in the gap is

$$(\Delta P)_{gap} = \frac{\beta^2}{2}(R_o^2 - r_o^2) + H_1 Q \ln\left(\frac{R_o}{r_o}\right) + H_2 Q^2 \left(\frac{1}{R_o^2} - \frac{1}{r_o^2}\right)$$

$$= 1{,}656 - 0.315\, Q + 0.13 \times 10^{-4}\, Q^2$$

Assuming that seal is open to inlet pressure on external side

$$(\Delta P)_{Imp} = (\Delta P)_{gap} + (\Delta P)_{seal}$$

which quantitatively is written

$$13{,}200 = 1656 - 0.315Q + 0.13 \times 10^{-4} Q^2 + .0122\, Q^2$$

or

$$.0122\, Q^2 - 0.315Q - 11{,}520 = 0$$

This may now be solved for Q. As the problem is about leakage, the negative root will be taken. The result is

$$Q = -964$$

To check the validity of the analysis for this problem, different terms can be compared in the expression of $\frac{dP}{dr}$ at inner and outer radii.

$$\frac{dP}{dr} = \beta^2 r + \frac{H_1 Q}{r} - \frac{2H_2 Q^2}{r^3}$$

Now at inner radius (r_o = 80)

$$\frac{Q}{r_o^2} \times \frac{H_1}{\beta^2} = .514$$

$$\frac{Q}{r_o^2} \times \frac{2H_2}{H_1} = .0877$$

And at outer radius (R_o = 200)

$$\frac{Q}{R_o^2} \times \frac{H_1}{\beta^2} = .0825$$

$$\frac{Q}{R_o^2} \times \frac{2H_2}{H_1} = .0141$$

As expected, it is found that succeeding terms decrease more rapidly at the outer radius but even at inner radius each term is smaller than previous term by enough to indicate that truncation error is reasonably small and this analysis is applicable.

Assuming that, $\beta_L = \beta_R$, $r_{oL} = r_{oR}$ and that there is leakage on one side only, the thrust can be evaluated from (6-4).

$$\text{Thrust} = 2\pi H_1 Q\left[\frac{r_o^2}{2}\ln\left(\frac{R_o}{r_o}\right) - \frac{R_o^2 - r_o^2}{4}\right] + 2\pi H_2 Q^2\left[\ln\left(\frac{R_o}{r_o}\right) - \frac{R_o^2 - r_o^2}{4R_o^2}\right]$$

$$= -2\pi H_1 Q(.528)\times 10^4 + 2\pi H_2 Q^2(.70)$$

$$= -1.141 \times 10^7$$

In dimensional form

Thrust = $-1.141 \times 10^{7} \times 2.18 \times 10^{-6}$ lb.

= 25 lb.

This value can be compared with the extreme case of infinite leakage as shown by Cooper [10] . The assumption of infinite leakage implies that the inflowing fluid tends to conserve the angular momentum it possessed upon entering the clearance space at the impeller outer radius. In this case pressure decreases more rapidly with the decreasing radius as the angular speed increases. So expression for thrust for infinite leakage on one side and no leakage on other side is,

$$\text{Thrust} = \frac{\Omega^2 \pi \bar{\rho} \beta^2 \bar{R}_o^4}{2} \left(2 \ln\left(\frac{R_o}{r_o}\right) - 1.5 - \frac{r_o^4}{R_o^4} + 2 \frac{r_o^2}{R_o^2} \right)$$

= 336 lb.

Actual leakage ratio for this problem

$\bar{Q} = 964 \times 6.3 \times 10^{-6}$ ft^3/sec

= 2.73 gpm

= 5.5% of total flow rate

If this centrifugal pump has an impeller with a shroud on one side only then the thrust will be larger. The force developed on the side of impeller where there is no shroud is calculated assuming the fluid to rotate at the impeller angular speed. This yields

$$F = 2\pi \int_{r_o}^{R_o} \left(P_o - \frac{R_o^2 - r^2}{2}\right) r\,dr$$

$$= 2\pi P_o \left(\frac{R_o^2 - r_o^2}{2}\right) - \left(\frac{2\pi}{8}\right)\left(R_o^2 - r_o^2\right)^2 \tag{6-6}$$

The thrust due to the shrouded side of the impeller is given by equation (6-3). The thrust for an impeller shrouded on one side only is the difference between (6-6) and (6-3). For $r_{oR} = r_{oL}$, this becomes

$$\text{Thrust} = -\frac{2\pi}{8}\left(R_o^2 - r_o^2\right)\beta^2 + \frac{2\pi}{8}\left(R_o^2 - r_o^2\right)^2 -$$

$$+ 2\pi H_1 Q \left[\frac{r_o^2}{2}\left(\ln\frac{R_o}{r_o}\right) - \frac{R_o^2 - r_o^2}{4}\right]$$

$$+ 2\pi H_2 Q \left[\ln\frac{R_o}{r_o} - \frac{R_o^2 - r_o^2}{4R_o^2}\right]$$

$$= 78.46 \times 10^7$$

In dimensional form for the calculated example the thrust is 1,710 lbs. in a direction opposite to the inlet flow.

Most of the axial force for a single shrouded impeller is because of the difference in average angular velocity on the two sides of the impeller. The effect of leakage is of the order of $\frac{25}{1710}$ or 1.5%.

VII. SUMMARY OF RESULTS

The problem of inflow between a stationary disk (wall) and a rotating disk has been solved by a linearization method. Similarity solutions are sought while end effects are neglected by taking infinite disks. The momentum and energy equations are simplified by assuming an axisymmetric laminar flow of a constant property fluid. The effect of inflow is found as a perturbation of the no-inflow base solution. Only two perturbation terms in the series are evaluated, since for other higher order terms, the inflow appears as a parameter in the differential equation. This precludes solution of the higher order equations unless the amount of inflow is specified a priori.

The solution of flow field and temperature field shed light on the influence of inflow on the pressure distribution, the shear stresses and the heat transfer rates at the disks. It is found that inflow increases the pressure gradient in the gap and so results in an increased axial thrust above that due to the basic flow. The effect becomes prominent for situations, vix., large leakage rate, large angular velocity of the disk or small viscosity.

Inflow also reduces the shear stresses on the rotating disk and increases the shear stress on the wall. So it has the effect of reducing frictional power loss on the rotating disk. Overall, the leakage has a negative influence on pump performance, as it carries with it some kinetic energy which is lost.

Another effect of inflow is on the recirculation region. The influence of leakage becomes large at small radius as the angular momentum

of the basic flow diminishes to zero as the axis is approached. This inflow creates a recirculation cell about the axis of rotation, with fluid coming inward at rotating disk instead being thrown out. This recirculation will slightly affect the power loss due to radial shear stresses.

A surprising influence of inflow is on the temperature field. For the case, wherein the wall is cooled to a constant temperature and the rotating disk is insulated, the disk temperature is independent of amount of inflow, although the heat transfer rate at the wall decreases. For the case where both the disk and the wall are cooled to the same temperature, the heat transfer rate at the wall decreased as expected.

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APPENDIX

Solutions of system-1 as listed in tables 1, 4, and 7 are directly taken from Cooper [10] results. They have to be modified in order to correspond to the co-ordinate system of this report.

Conversion is as follows:

Position relative to wall,

Cooper's notation	Current notation
F	- F
FP	FP
FPP	- FPP
GP	G
GPP	- GP

Position relative to Disk

Cooper's notation	Current notation
F	F
FP	FP
FPP	FPP
GP	G
GPP	GP

Here the number of P's represents the order of the derivative with respect to the argument.

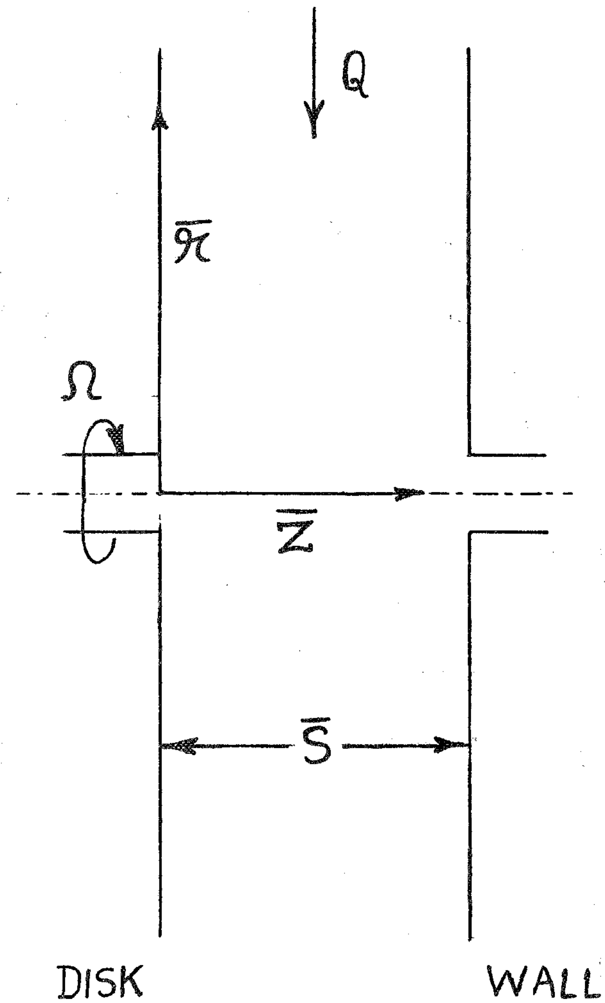

Figure 1

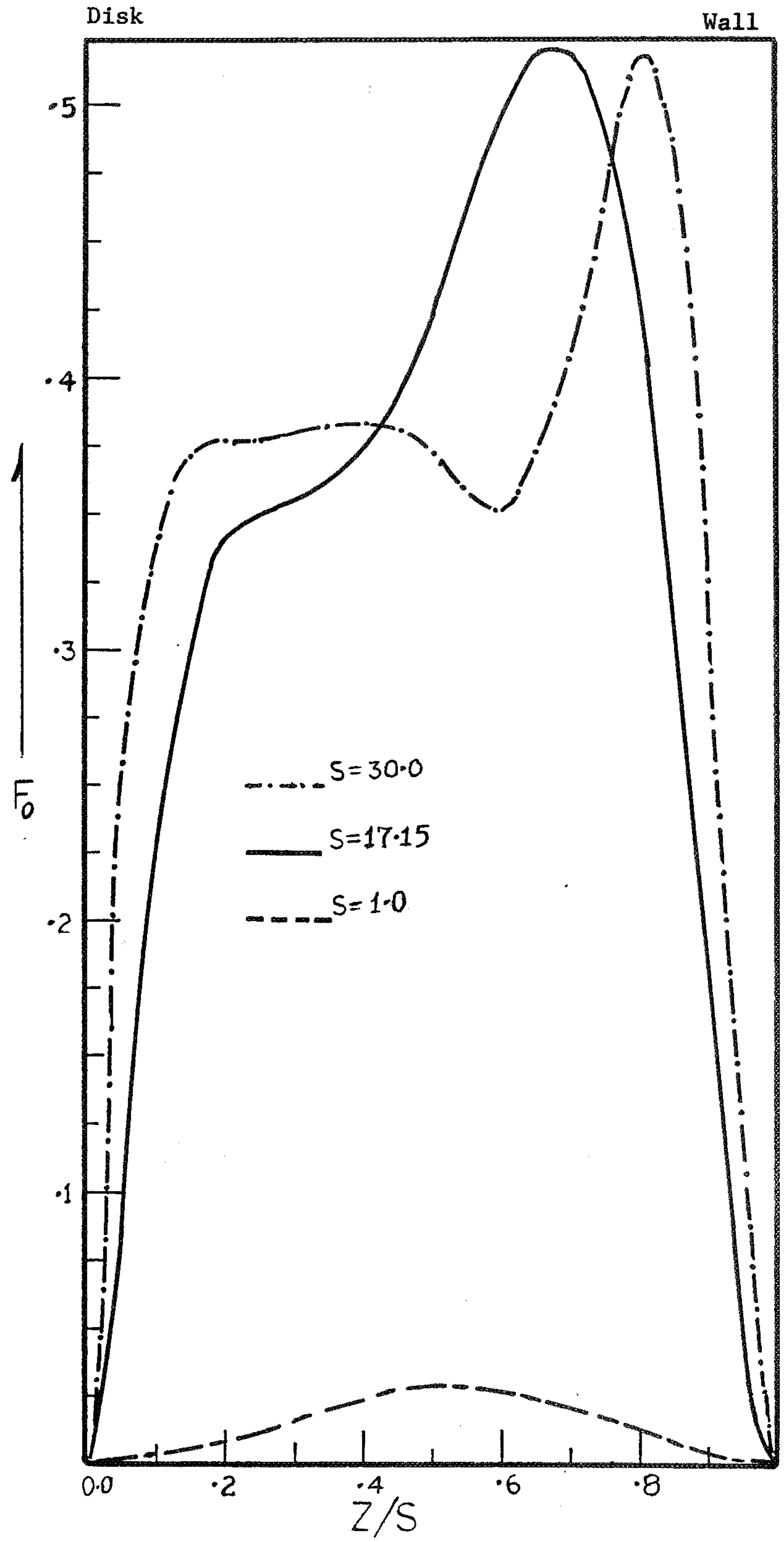

Figure 2

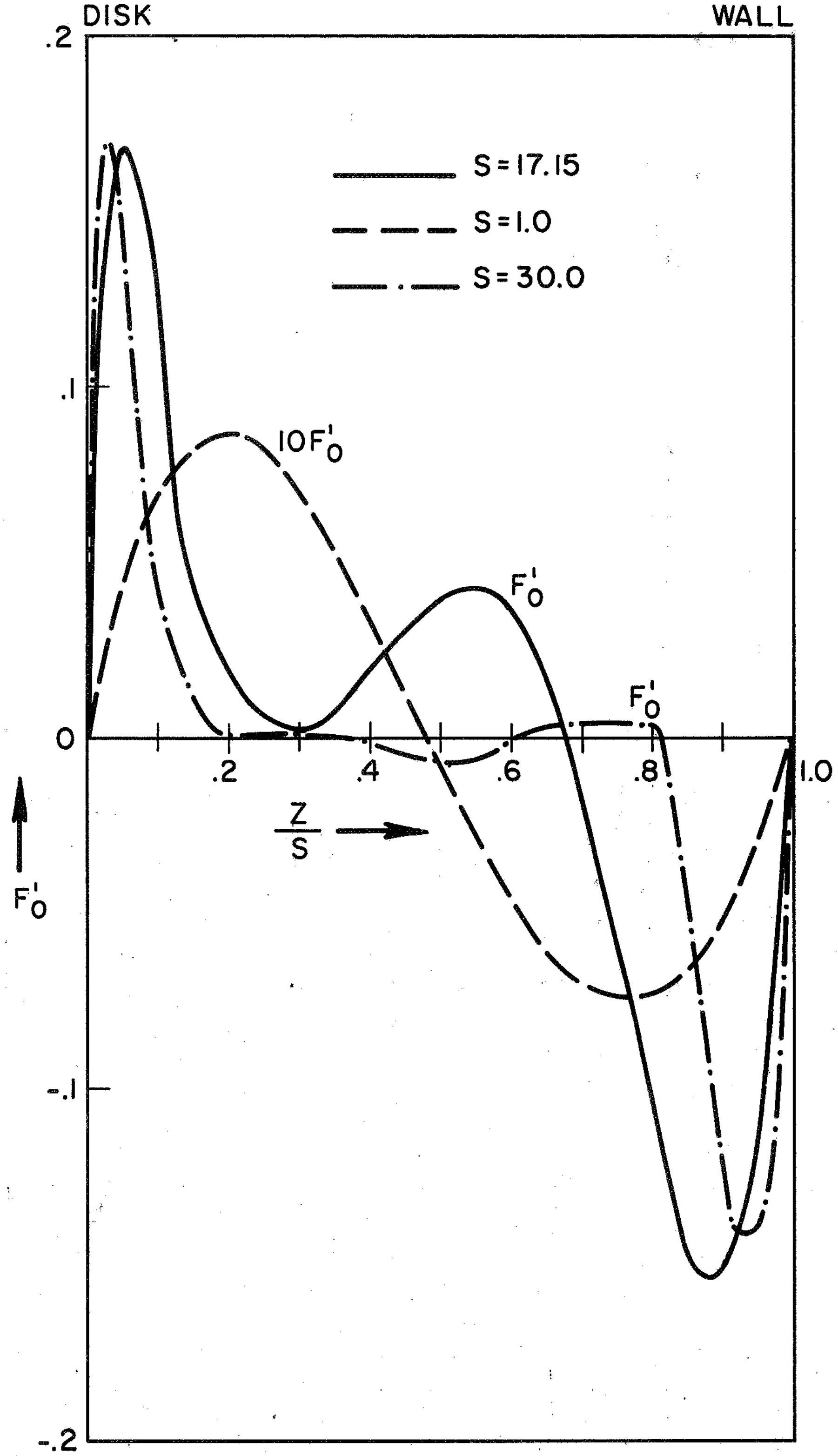

Figure 3

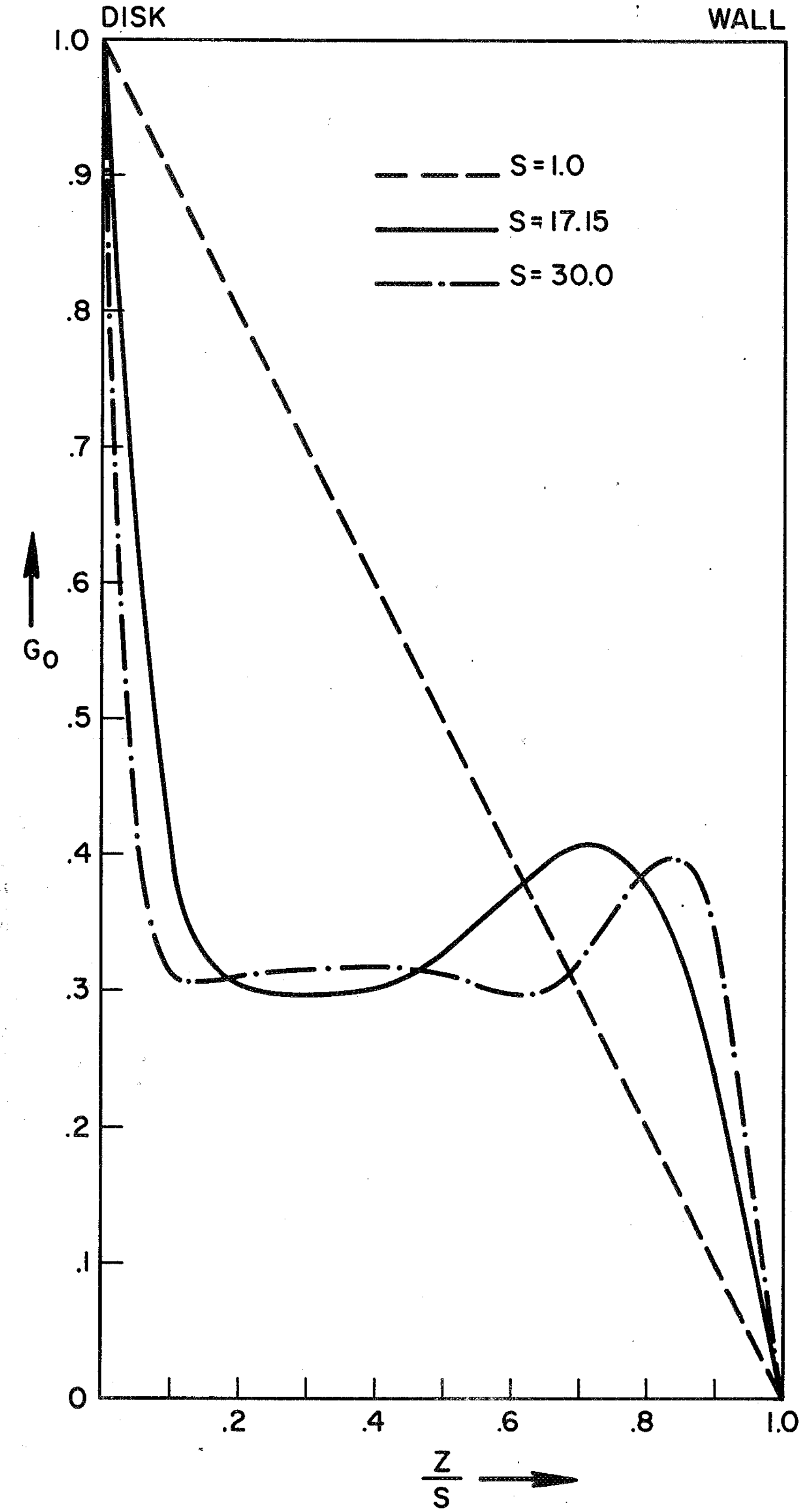

Figure 4

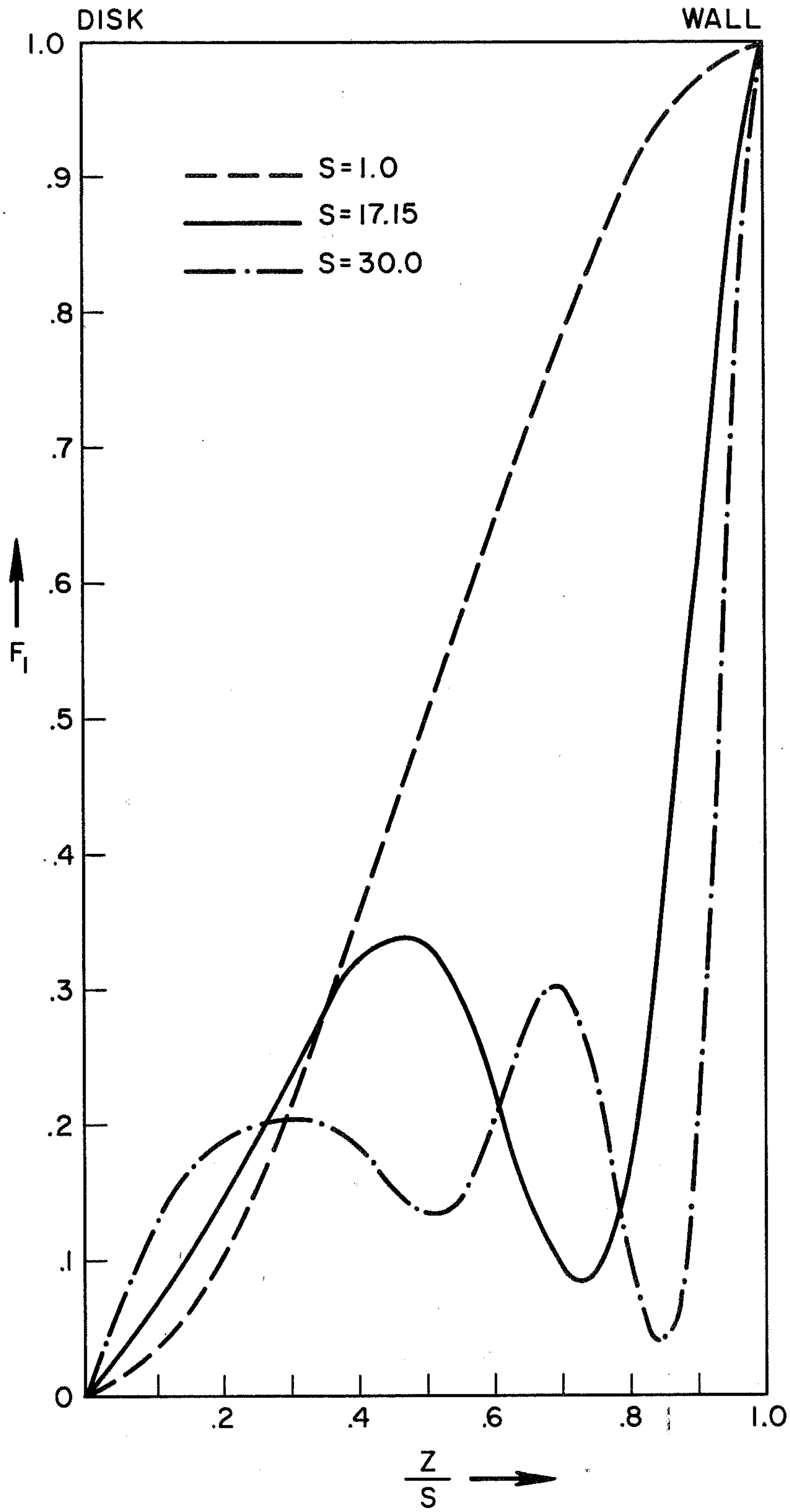

Figure 5

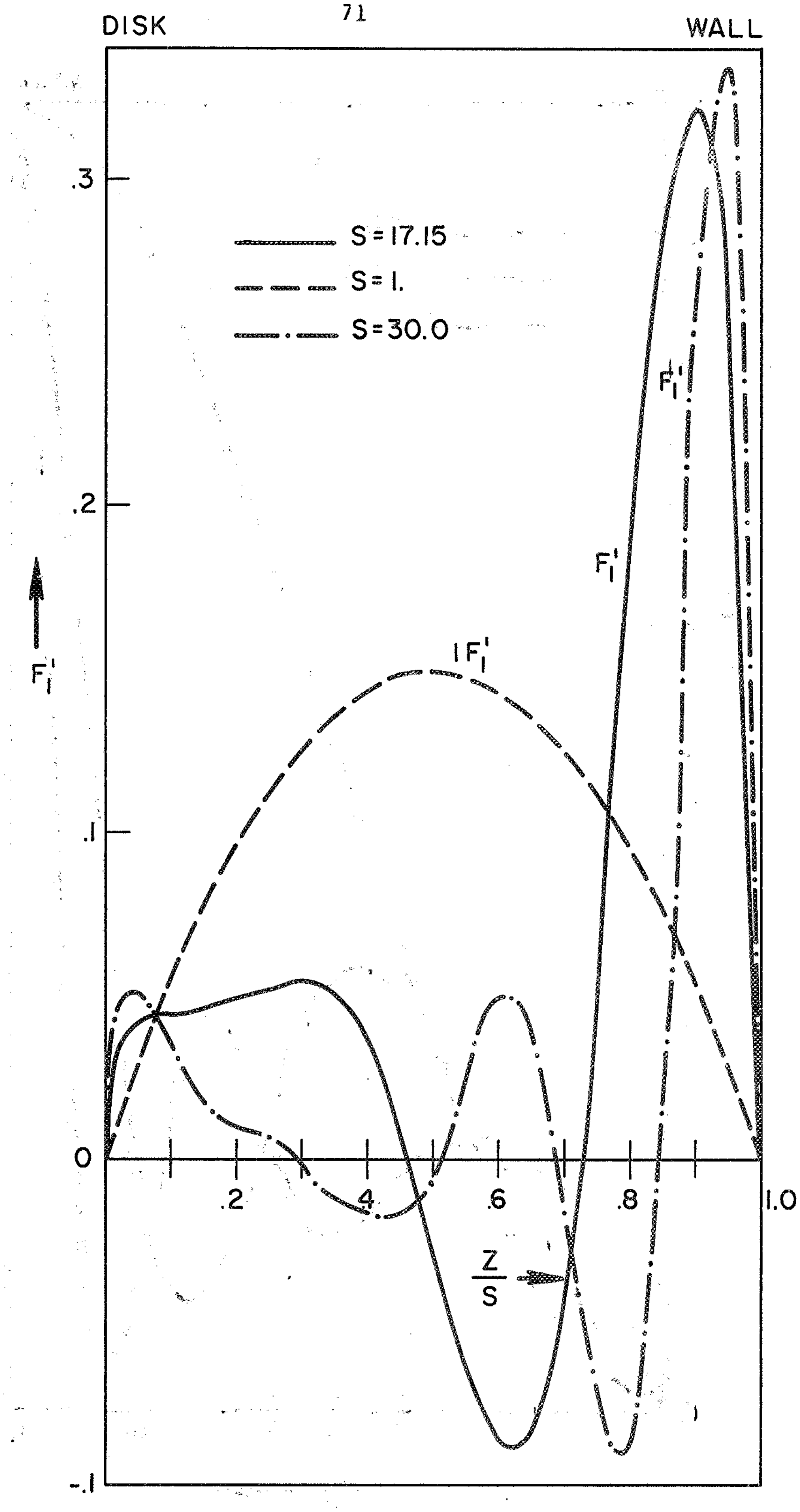

Figure 6

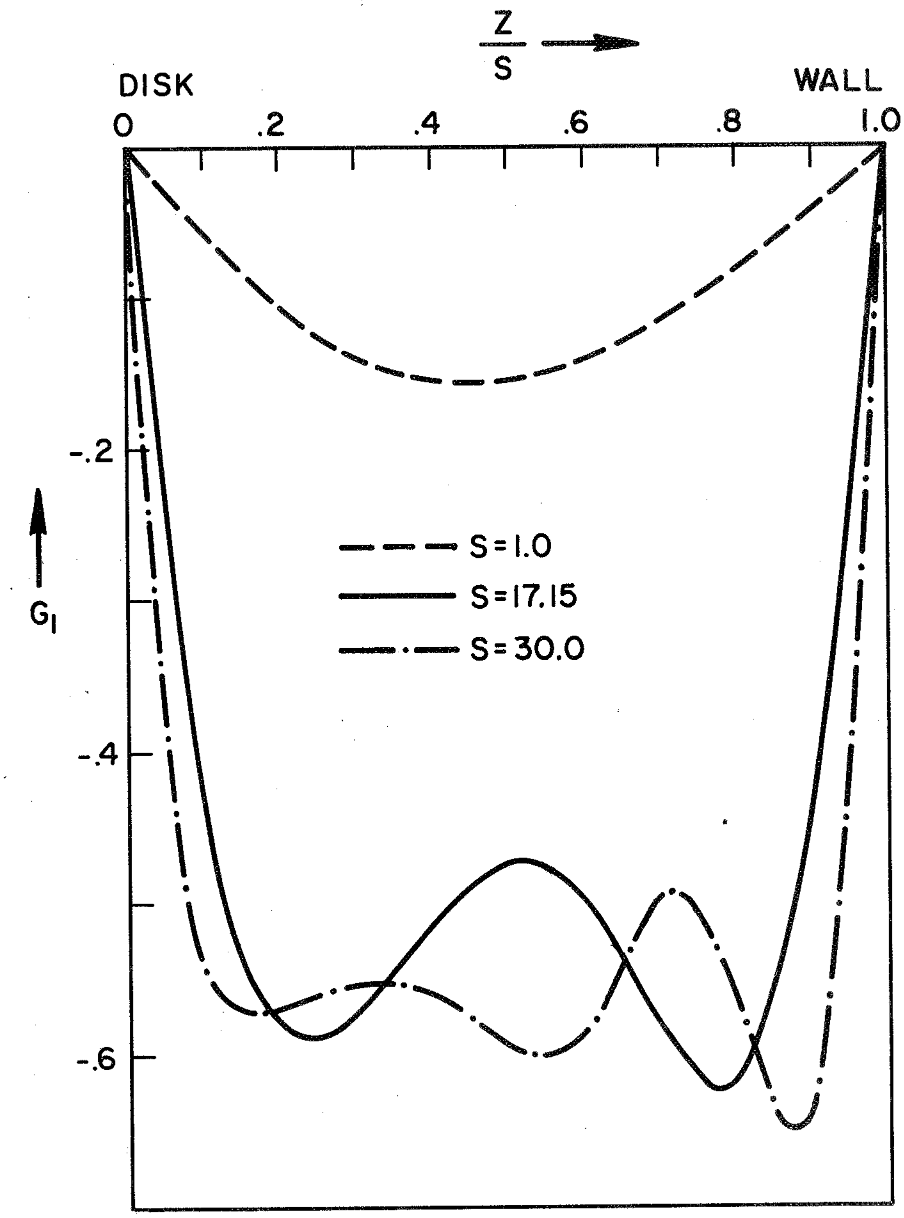

Figure 7

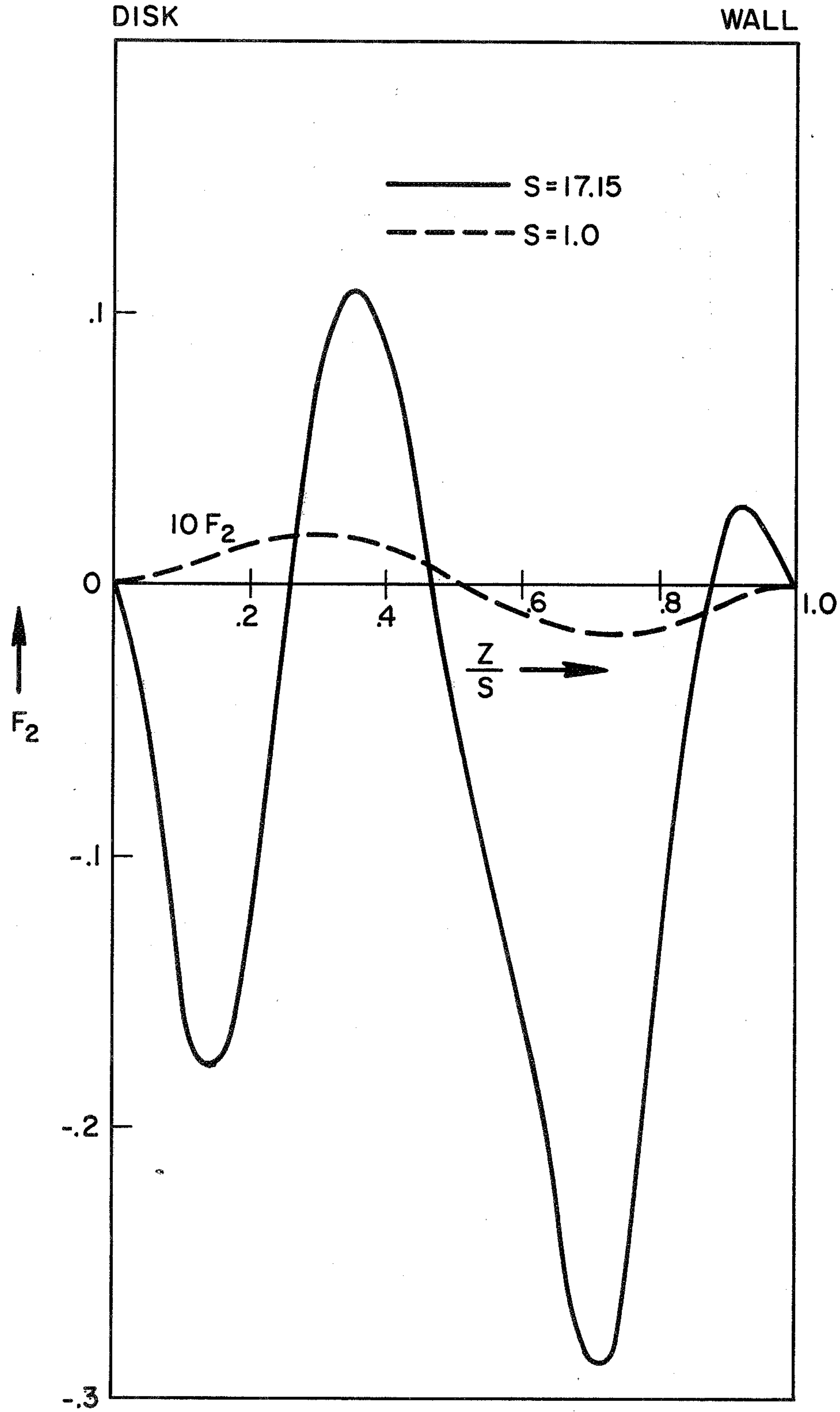

Figure 8

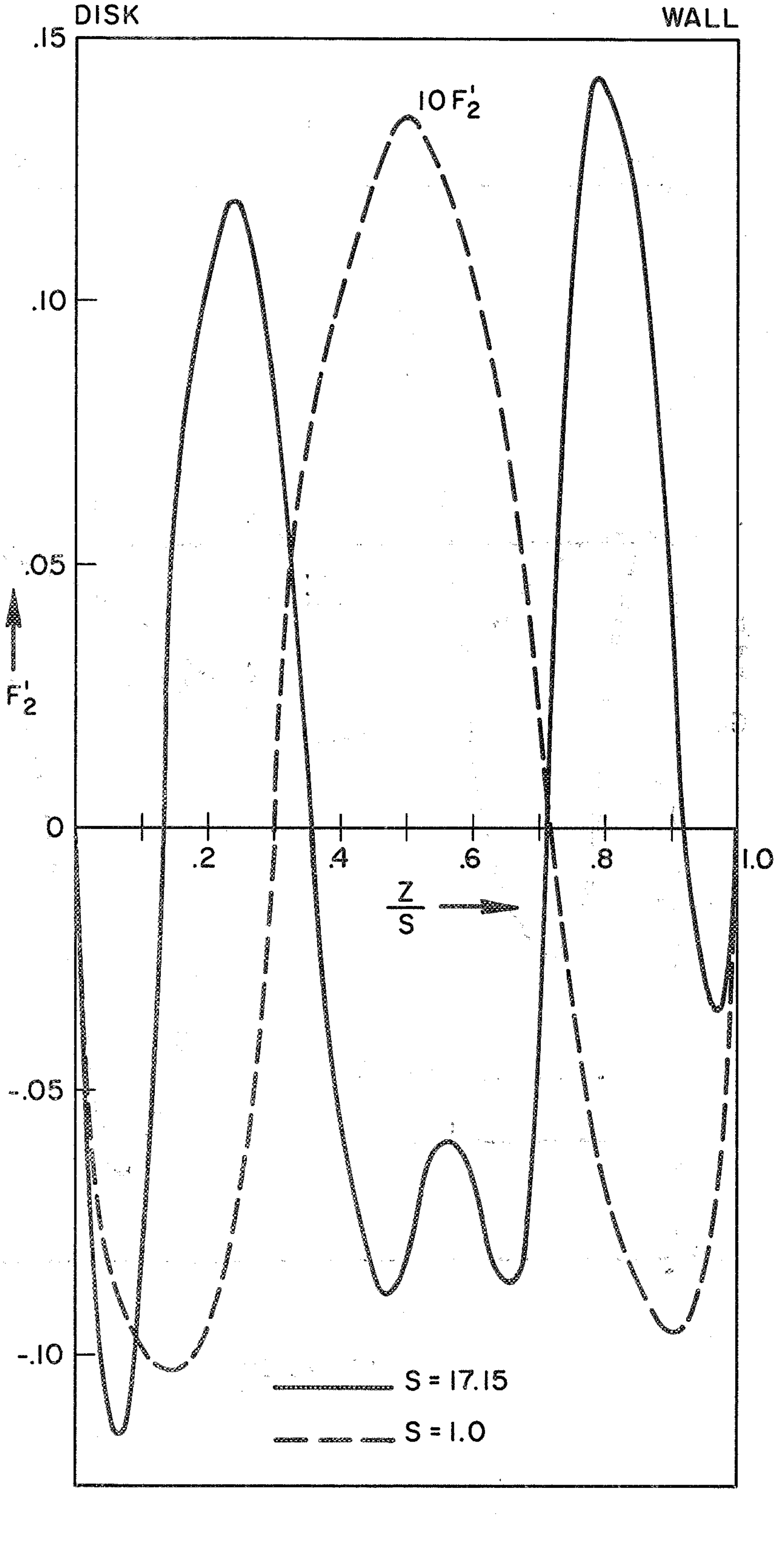

Figure 9

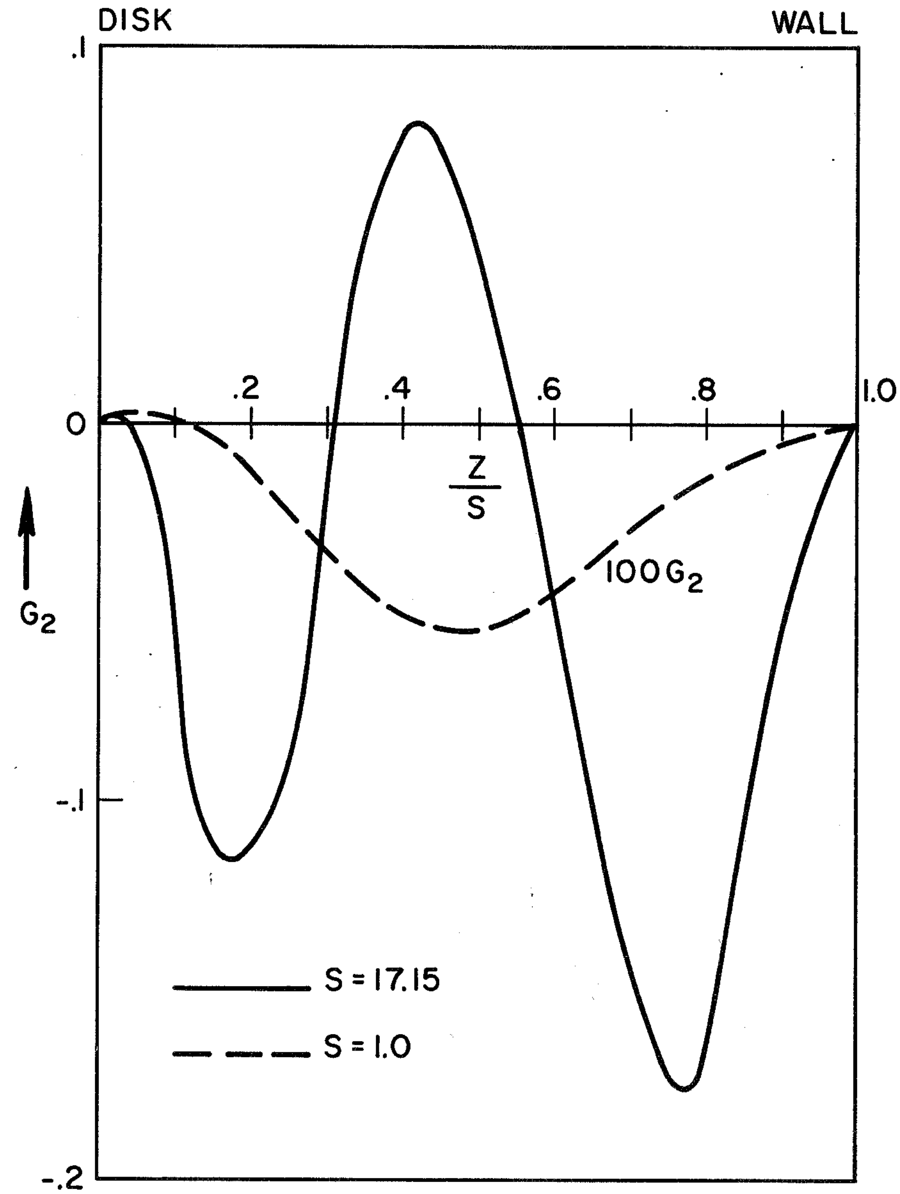

Figure 10

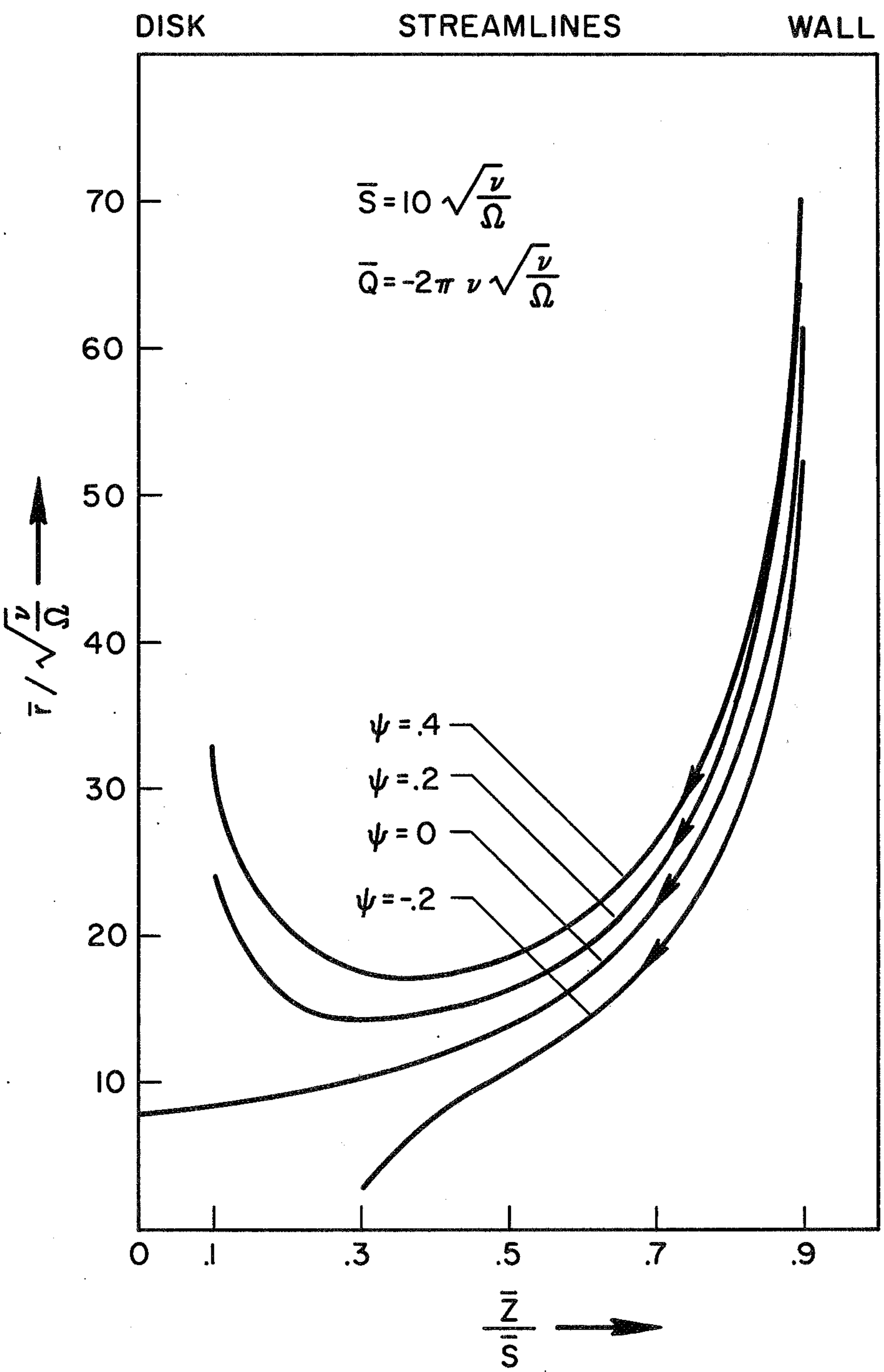
DISK
STREAMLINES
WALL
$\bar{S} = 10\sqrt{\frac{\nu}{\Omega}}$
$\bar{Q} = -2\pi\,\nu\sqrt{\frac{\nu}{\Omega}}$
70
60
50
40
30
20
10
$\bar{r}/\sqrt{\frac{\nu}{\Omega}}$
$\psi = .4$
$\psi = .2$
$\psi = 0$
$\psi = -.2$
0
.1
.3
.5
.7
.9
$\frac{\bar{Z}}{\bar{S}}$

Figure 11

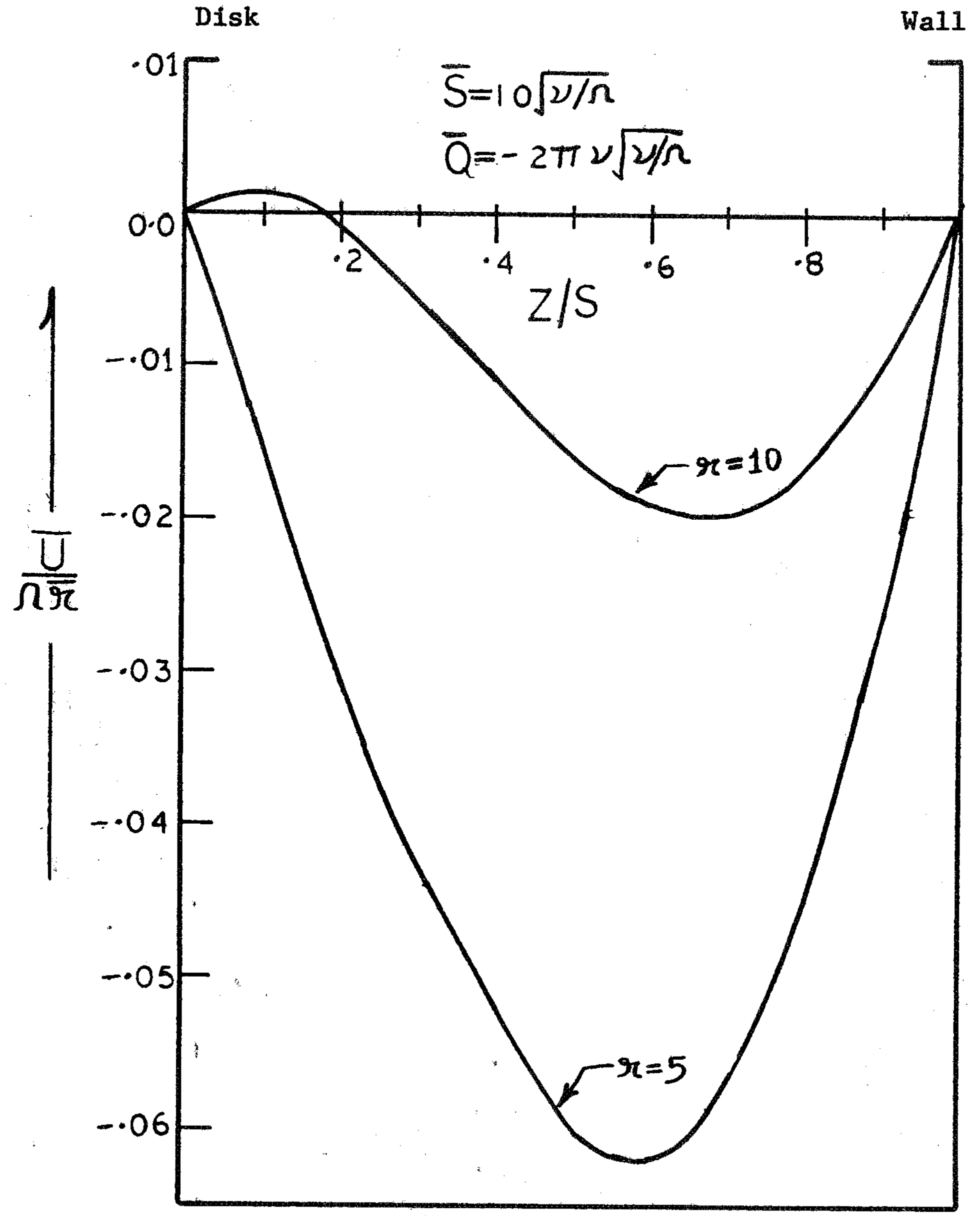
Disk
Wall
$\bar{S}=10\sqrt{\nu/\Omega}$
$\bar{Q}=-2\pi\nu\sqrt{\nu/\Omega}$
·01
0·0
-·01
-·02
-·03
-·04
-·05
-·06
·2
·4
·6
·8
Z/S
$\frac{\bar{U}}{\Omega\bar{r}}$
r=10
r=5

Figure 12

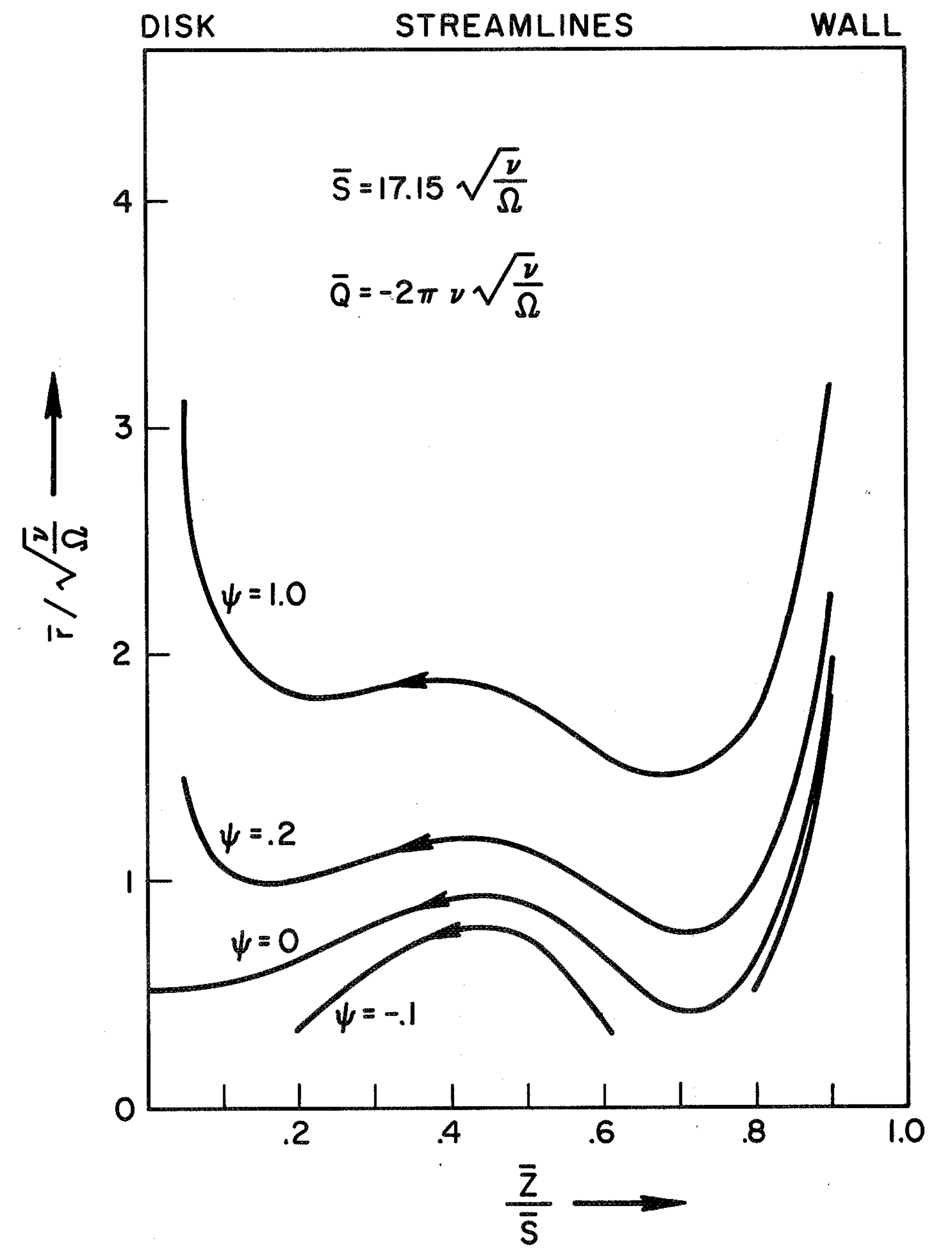
DISK
STREAMLINES
WALL
$\bar{S} = 17.15\sqrt{\frac{\nu}{\Omega}}$
$\bar{Q} = -2\pi\nu\sqrt{\frac{\nu}{\Omega}}$
$\bar{r}/\sqrt{\frac{\nu}{\Omega}}$
$\psi = 1.0$
$\psi = .2$
$\psi = 0$
$\psi = -.1$
0
1
2
3
4
.2
.4
.6
.8
1.0
$\frac{\bar{Z}}{\bar{S}}$

Figure 13

TABLE 1*

S = 1

System 1

VELOCITY AND SHEAR STRESS PROFILES

POSITION RELATIVE TO WALL			F	FP	FPP
I	Z/S	ETA / NUE	W	GP	GPP
1	.00000000	.00000	.00000000	.00000000	-.06622170
		1.00000	.00000000	.00000000	.99836537
5	.10000000	.10000	-.00028151	-.00513739	-.03669207
		1.00000	.00056301	.09983608	.99834828
9	.20000000	.20000	-.00093159	-.00740480	-.00915407
		1.00000	.00186319	.19966740	.99826715
13	.30000000	.30000	-.00167654	-.00710092	.01440230
		1.00000	.00335308	.29948844	.99815668
17	.40000000	.40000	-.00228244	-.00472341	.03198670
		1.00000	.00456488	.39930169	.99813639
21	.50000000	.50000	-.00257511	-.00096908	.04160583
		1.00000	.00515022	.49912422	.99837088

POSITION RELATIVE TO DISK			F	FP	FPP
I	Z/S	ETA / NUE	W	GP	GPP
1	.00000000	.00000	.00000000	.00000000	.10041434
		1.00000	.00000000	1.00000000	-1.00398336
5	.10000000	.10000	.00039337	.00686076	.03997902
		1.00000	-.00078674	.89962900	-1.00321949
9	.20000000	.20000	.00120158	.00859761	-.00240228
		1.00000	-.00240316	.79938097	-1.00169221
13	.30000000	.30000	.00199911	.00691441	-.02876032
		1.00000	-.00399823	.69928993	-1.00017095
17	.40000000	.40000	.00252071	.00331209	-.04112176
		1.00000	-.00504142	.59933382	-.99902977
21	.50000000	.50000	.00264107	-.00091066	-.04150427
		1.00000	-.00528214	.49946782	-.99836792

* Refer to Appendix.

TABLE 2

S = 1

System 2

	F	FP	FPP	G	GP	FPPP	GPP	H1	FTA POS	FRACT Z/S
WALL	1.00000	.00000	-6.02089	.00000	.39887	-12.12436	.00000	-12.12436		
DISK	.00000	.00000	5.99943	.00000	-.59945	-12.12436	.00000	-12.12436	.00000	.00000
	.02798	.53951	4.79244	-.05812	-.54674	-12.02357	.97115	-12.12436	.10000	.10000
	.10389	.95875	3.59348	-.10684	-.41840	-11.96218	1.53383	-12.12436	.20000	.20000
	.21574	1.25835	2.39877	-.14051	-.25114	-11.93743	1.76090	-12.12436	.30000	.30000
	.35158	1.43854	1.20499	-.15679	-.07495	-11.94250	1.72471	-12.12436	.40000	.40000
	.49947	1.49929	.00957	-.15597	.08745	-11.96861	1.49724	-12.12436	.50000	.50000
	.64743	1.44033	-1.18911	-.14027	.22044	-12.00691	1.14925	-12.12436	.60000	.60000
	.78354	1.26132	-2.39190	-.11314	.31574	-12.04857	.75444	-12.12436	.70000	.70000
	.89573	.96183	-3.59870	-.07844	.37217	-12.08632	.38340	-12.12436	.80000	.80000
	.97197	.54148	-4.89882	-.03982	.39564	-12.11370	.10790	-12.12436	.90000	.90000
	1.00000	.00000	-6.02089	.00000	.39887	-12.12436	.00000	-12.12436	1.00000	1.00000

TABLE 3

S = 1

System 3

WALL	F	FP	FPP	G	GP	FPPP	GPP	H2		
	.00000	.00000	.17330	.00000	.00053	1.55485	.00000	-.77742		
									ETA POS FRACT	
	.00000	.00000	-.17517	.00000	.00053	1.55485	.00000	-.77742	.00000	.00000
	-.00062	-.00992	-.02811	.00001	-.00060	1.25982	-.01660	-.77742	.10000	.10000
	-.00157	-.00741	.06750	-.00013	-.00199	.62472	-.00871	-.77742	.20000	.20000
	-.00189	.00132	.09605	-.00034	-.00213	-.03698	.00565	-.77742	.30000	.30000
	-.00131	.00984	.06657	-.00051	-.00105	-.50788	.01443	-.77742	.40000	.40000
	-.00009	.01355	.00403	-.00054	.00043	-.76653	.01372	-.77742	.50000	.50000
									ETA POS FRACT	
	-.00009	.01355	.00403	-.00054	.00043	-.76653	.01372	-.77742	.50000	.50000
	.00117	.01041	-.06447	-.00044	.00145	-.56307	.00597	-.77742	.60000	.60000
	.00181	.00189	-.09742	-.00028	.00160	-.05402	-.00249	-.77742	.70000	.70000
	.00153	-.00703	-.06952	-.00014	.00113	.62493	-.00586	-.77742	.80000	.80000
	.00061	-.00973	.02622	-.00006	.00064	1.26062	-.00316	-.77742	.90000	.90000
	.00000	.00000	.17330	.00000	.00053	1.55485	.00000	-.77742	1.00000	1.00000

TABLE 4

S = 17.15

System 1

VELOCITY AND SHEAR STRESS PROFILES

POSITION RELATIVE TO WALL I	Z/S	ETA / NUE	F / W	FP / GP	FPP / GPP
1	.00000000	.00000	.00000000	.00000000	-.16978390
		1.00000	.00000000	.00000000	.13930740
5	.00583090	.10000	-.00083195	-.01646947	-.15960871
		1.00000	.00166391	.01393055	.13929976
9	.01166181	.20000	-.00326002	-.03192297	-.14947037
		1.00000	.00652005	.02785848	.13924810
13	.01749271	.30000	-.00718286	-.04636591	-.13940278
		1.00000	.01436572	.04177740	.13911357
17	.02332361	.40000	-.01249981	-.05980692	-.12943679
		1.00000	.02499961	.05567731	.13886272
21	.02915452	.50000	-.01911123	-.07225759	-.11960044
		1.00000	.03822246	.06954512	.13846736
25	.03498542	.60000	-.02691879	-.08373218	-.10991918
		1.00000	.05383757	.08336518	.13790420
29	.04081633	.70000	-.03582568	-.09424736	-.10041599
		1.00000	.07165136	.09711974	.13715462
33	.04664723	.80000	-.04573690	-.10382201	-.09111163
		1.00000	.09147381	.11078941	.13620425
37	.05247813	.90000	-.05655942	-.11247694	-.08202477
		1.00000	.11311885	.12435355	.13504264
41	.05830904	1.00000	-.06820239	-.12023477	-.07317213
		1.00000	.13640478	.13779067	.13366293
45	.06413994	1.10000	-.08057728	-.12711968	-.06456860
		1.00000	.16115456	.15107874	.13206149
49	.06997084	1.20000	-.09359808	-.13315724	-.05622741
		1.00000	.18719615	.16419555	.13023756
53	.07580175	1.30000	-.10716137	-.13837429	-.04816017
		1.00000	.21436275	.17711889	.12819295
57	.08163265	1.40000	-.12124651	-.14279874	-.04037700
		1.00000	.24249303	.18982691	.12593174

Table 4 (continued)

	Z/S	ETA	F	FP	FPP
		NUE	W	GP	GPP
61	.08746356	1.50000	-.13571566	-.14645945	-.03288660
		1.00000	.27143133	.20229822	.12345996
65	.09475219	1.62500	-.15425643	-.15000656	-.02394641
		1.00000	.30851287	.21752306	.12008608
69	.10641399	1.82500	-.18464592	-.15344492	-.01064525
		1.00000	.36929183	.24095125	.11407560
73	.11807580	2.02500	-.21546543	-.15434879	.00139452
		1.00000	.43093085	.26310853	.10739096
77	.12973761	2.22500	-.24623340	-.15297224	.01215793
		1.00000	.49246681	.28386947	.10013397
81	.14139941	2.42500	-.27651932	-.14957070	.02164569
		1.00000	.55303864	.30313104	.09241394
85	.15306123	2.62500	-.30594362	-.14439803	.02987272
		1.00000	.61188723	.32081167	.08434344
89	.16472303	2.82500	-.33417711	-.13770384	.03686670
		1.00000	.66835423	.33685255	.07603490
93	.17638484	3.02500	-.36093993	-.12973098	.04266660
		1.00000	.72187985	.35121708	.06759783
97	.18804665	3.22500	-.38599990	-.12071356	.04732130
		1.00000	.77199978	.36389010	.05913673
101	.19970845	3.42500	-.40917064	-.11087500	.05088818
		1.00000	.81834126	.37487671	.05074943
105	.21137026	3.62500	-.43030926	-.10042652	.05343176
		1.00000	.86061852	.38420080	.04252587
109	.22303207	3.82500	-.44931379	-.08956584	.05502234
		1.00000	.89862758	.39190340	.03454722
113	.23469388	4.02500	-.46612036	-.07847614	.05573471
		1.00000	.93224072	.39804083	.02688534
117	.24635568	4.22500	-.48070021	-.06732530	.05564689
		1.00000	.96140041	.40268282	.01960241
121	.25801750	4.42500	-.49305658	-.05626539	.05483896
		1.00000	.98611315	.40591057	.01275091
125	.26967930	4.62500	-.50322152	-.04543235	.05339192
		1.00000	1.00644304	.40781482	.00637365
129	.28134111	4.82500	-.51125264	-.03494583	.05138671
		1.00000	1.02250528	.40849390	.00050401
133	.29300292	5.02500	-.51722990	-.02490953	.04890320
		1.00000	1.03445980	.40805191	-.00483369
137	.30466472	5.22500	-.52125236	-.01541123	.04601937
		1.00000	1.04250471	.40659695	-.00962374
141	.31632653	5.42500	-.52343512	-.00652343	.04281059
		1.00000	1.04687025	.40423945	-.01385847
145	.32798834	5.62500	-.52390631	.00169615	.03934890
		1.00000	1.04781261	.40109061	-.01753768
149	.33965015	5.82500	-.52280416	.00920382	.03570247
		1.00000	1.04560831	.39726099	-.02066797
153	.35131195	6.02500	-.52027430	.01596909	.03193516
		1.00000	1.04054861	.39285918	-.02326206
157	.36297376	6.22500	-.51646724	.02197378	.02810614
		1.00000	1.03293449	.38799071	-.02533809
161	.37463557	6.42500	-.51153597	.02721105	.02426957
		1.00000	1.02307194	.38275696	-.02691893
165	.38629737	6.62500	-.50563377	.03168439	.02047448
		1.00000	1.01126754	.37725436	-.02803148

Table 4 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
169	.39795918	6.82500	-.49891232	.03540654	.01676461
		1.00000	.99782463	.37157358	-.02870598
173	.40962099	7.02500	-.49151986	.03839849	.01317840
		1.00000	.98303971	.36579899	-.02897535
177	.42128280	7.22500	-.48359974	.04068837	.00974904
		1.00000	.96719947	.36000812	-.02887459
181	.43294460	7.42500	-.47528905	.04231044	.00650454
		1.00000	.95057809	.35427139	-.02844018
185	.44460642	7.62500	-.46671747	.04330406	.00346798
		1.00000	.93343494	.34865180	-.02770950
189	.45626822	7.82500	-.45800643	.04371272	.00065765
		1.00000	.91601285	.34320482	-.02672038
193	.46793003	8.02500	-.44926827	.04358313	-.00191260
		1.00000	.89853654	.33797836	-.02551060
197	.47959184	8.22500	-.44060579	.04296434	-.00423309
		1.00000	.88121157	.33301280	-.02411747
201	.49125364	8.42500	-.43211178	.04190695	-.00629790
		1.00000	.86422355	.32834116	-.02257749

POSITION RELATIVE TO DISK

I	Z/S	ETA NUE	F W	FP GP	FPP GPP
1	.00000000	.00000	.00000000	.00000000	.46893006
		1.00000	.00000000	1.00000000	-.52032585
5	.00583090	.10000	.00219924	.04257319	.38422060
		1.00000	-.00439848	.94811452	-.51600131
9	.01166181	.20000	.00824887	.07717133	.30932511
		1.00000	-.01649774	.89704304	-.50436644
13	.01749271	.30000	.01739939	.10474363	.24359099
		1.00000	-.03479877	.84742591	-.48718191
17	.02332361	.40000	.02899287	.12617185	.18632621
		1.00000	-.05798574	.79974245	-.46591562
21	.02915452	.50000	.04245608	.14226758	.13682279
		1.00000	-.08491217	.75433807	-.44178379
25	.03498542	.60000	.05729340	.15377159	.09437520
		1.00000	-.11458681	.71144767	-.41578647
29	.04081633	.70000	.07307978	.16135484	.05829492
		1.00000	-.14615957	.67121562	-.38873845
33	.04664723	.80000	.08945386	.16562071	.02702009
		1.00000	-.17890773	.63371297	-.36129631
37	.05247813	.90000	.10611138	.16710810	.00262395
		1.00000	-.21222275	.59895198	-.33398202
41	.05830904	1.00000	.12279887	.16629518	-.01818066
		1.00000	-.24559774	.56689860	-.30720358
45	.06413994	1.10000	.13930784	.16360350	-.03503817
		1.00000	-.27861569	.53748292	-.28127296
49	.06997084	1.20000	.15546931	.15940239	-.04844896
		1.00000	-.31093861	.51060796	-.25642177
53	.07580175	1.30000	.17114877	.15401334	-.05886927
		1.00000	-.34229754	.48615703	-.23281486
57	.08163265	1.40000	.18624169	.14771441	-.06671207
		1.00000	-.37248337	.46399981	-.21056198

Table 4 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
61	.08746356	1.50000	.20066932	.14074443	-.07234870
		1.00000	-.40133864	.44399730	-.18972792
65	.09475219	1.62500	.21768384	.13139559	-.07679396
		1.00000	-.43536769	.42180537	-.16572101
69	.10641399	1.82500	.24240257	.11571283	-.07919390
		1.00000	-.48480514	.39213026	-.13197302
73	.11807580	2.02500	.26396740	.09999579	-.07741290
		1.00000	-.52793480	.36865116	-.10369382
77	.12973761	2.22500	.28244478	.08492917	-.07289746
		1.00000	-.56488957	.35031998	-.08040302
81	.14139941	2.42500	.29801174	.07094661	-.06672531
		1.00000	-.59602348	.33619578	-.06152399
85	.15306123	2.62500	.31091256	.05829675	-.05968025
		1.00000	-.62182511	.32545658	-.04645268
89	.16472303	2.82500	.32142720	.04709537	-.05231662
		1.00000	-.64285440	.31740016	-.03460116
93	.17638484	3.02500	.32984896	.03736582	-.04501291
		1.00000	-.65969792	.31143798	-.02542350
97	.18804665	3.22500	.33646917	.02906959	-.03801538
		1.00000	-.67293833	.30708534	-.01842918
101	.19970845	3.42500	.34156723	.02212918	-.03147294
		1.00000	-.68313446	.30394955	-.01318824
105	.21137026	3.62500	.34540459	.01644472	-.02546434
		1.00000	-.69080918	.30171791	-.00933114
109	.22303207	3.82500	.34822150	.01190580	-.02001927
		1.00000	-.69644298	.30014579	-.00654533
113	.23469388	4.02500	.35023575	.00839962	-.01513436
		1.00000	-.70047151	.29904583	-.00457014
117	.24635568	4.22500	.35164285	.00581632	-.01078520
		1.00000	-.70328570	.29827810	-.00319079
121	.25801750	4.42500	.35261688	.00405226	-.00693512
		1.00000	-.70523375	.29774152	-.00223236
125	.26967930	4.62500	.35331198	.00301183	-.00354158
		1.00000	-.70662396	.29736654	-.00155400
129	.28134111	4.82500	.35386404	.00260813	-.00056062
		1.00000	-.70772808	.29710878	-.00104354
133	.29300292	5.02500	.35439245	.00276291	.00205006
		1.00000	-.70878489	.29694385	-.00061284
137	.30466472	5.22500	.35500175	.00340606	.00432917
		1.00000	-.71000349	.29686292	-.00019364
141	.31632653	5.42500	.35578322	.00447475	.00631067
		1.00000	-.71156644	.29686911	.00026592
145	.32798834	5.62500	.35681623	.00591240	.00802315
		1.00000	-.71363246	.29697455	.00080420
149	.33965015	5.82500	.35816935	.00766762	.00948970
		1.00000	-.71633869	.29719789	.00144848
153	.35131195	6.02500	.35990129	.00969307	.01072798
		1.00000	-.71980257	.29756224	.00221689
157	.36297376	6.22500	.36206163	.01194445	.01175061
		1.00000	-.72412326	.29809363	.00312002
161	.37463557	6.42500	.36469131	.01437948	.01256557
		1.00000	-.72938261	.29881952	.00416210
165	.38629737	6.62500	.36782293	.01695710	.01317676
		1.00000	-.73564585	.29976767	.00534203

Table 4 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
169	.39795918	6.82500 1.00000	.37148094 -.74296188	.01963664 .30096515	.01358462 .00665411
173	.40962099	7.02500 1.00000	.37568165 -.75136331	.02237723 .30243748	.01378667 .00808863
177	.42128280	7.22500 1.00000	.38043313 -.76086626	.02513727 .30420789	.01377819 .00963230
181	.43294460	7.42500 1.00000	.38573502 -.77147003	.02787405 .30629659	.01355282 .01126863
185	.44460642	7.62500 1.00000	.39157826 -.78315653	.03054345 .30872022	.01310317 .01297817
189	.45626822	7.82500 1.00000	.39794487 -.79588973	.03309984 .31149127	.01242141 .01473873
193	.46793003	8.02500 1.00000	.40480752 -.80961505	.03549603 .31461748	.01149983 .01652561
197	.47959184	8.22500 1.00000	.41212936 -.82425871	.03768332 .31810146	.01033145 .01831171
201	.49125364	8.42500 1.00000	.41986360 -.83972719	.03961176 .32194016	.00891051 .02006776

TABLE 5

S = 17.15

System 2

	F	FP	FPP	G	GP	FPPP	GPP	H1	ETA POS	FRACT Z/S
WALL	1.00000	.00000	-.44716	.00000	.25936	-.34589	.00000	-.34589		
DISK	.00000	.00000	.14066	.00000	-.31563	-.34589	.00000	-.34589	.00000	.00000
	.00065	.01244	.10912	-.03152	-.31433	-.28660	.02497	-.34589	.10000	.00583
	.00239	.02201	.08311	-.06279	-.31082	-.23461	.04461	-.34589	.20000	.01166
	.00497	.02922	.06197	-.09362	-.30555	-.18937	.06016	-.34589	.30000	.01749
	.00817	.03454	.04503	-.12386	-.29389	-.15040	.07256	-.34589	.40000	.02332
	.01183	.03835	.03170	-.15336	-.29111	-.11720	.08258	-.34589	.50000	.02915
	.01580	.04098	.02141	-.18205	-.28244	-.08931	.09066	-.34589	.60000	.03499
	.02000	.04272	.01368	-.20983	-.27303	-.06621	.09725	-.34589	.70000	.04082
	.02433	.04379	.00803	-.23663	-.26303	-.04741	.10256	-.34589	.80000	.04665
	.02874	.04438	.00407	-.26242	-.25255	-.03240	.10676	-.34589	.90000	.05248
	.03319	.04465	.00144	-.28713	-.24171	-.02069	.10998	-.34589	1.00000	.05831
	.03766	.04470	-.00016	-.31075	-.23059	-.01180	.11230	-.34589	1.10000	.06414
	.04213	.04464	-.00100	-.33324	-.21928	-.00527	.11377	-.34589	1.20000	.06997
	.04859	.04452	-.00128	-.35460	-.20786	-.00067	.11444	-.34589	1.30000	.07580
	.05103	.04440	-.00119	-.37481	-.19641	.00238	.11436	-.34589	1.40000	.08163
	.05547	.04429	-.00085	-.39389	-.18501	.00422	.11358	-.34589	1.50000	.08746
	.06100	.04422	-.00024	-.41613	-.17092	.00526	.11172	-.34589	1.62500	.09475
	.06984	.04428	.00083	-.44811	-.14902	.00514	.10697	-.34589	1.82500	.10641
	.07872	.04454	.00175	-.47581	-.12824	.00400	.10055	-.34589	2.02500	.11806
	.08767	.04496	.00242	-.49950	-.10887	.00271	.09301	-.34589	2.22500	.12974
	.09672	.04549	.00285	-.51947	-.09108	.00169	.08488	-.34589	2.42500	.14140
	.10587	.04609	.00312	-.53604	-.07493	.00108	.07660	-.34589	2.62500	.15306
	.11516	.04674	.00331	-.54955	-.06043	.00084	.06852	-.34589	2.82500	.16472
	.12457	.04742	.00347	-.56032	-.04750	.00083	.06087	-.34589	3.02500	.17638
	.13413	.04813	.00365	-.56865	-.03604	.00090	.05381	-.34589	3.22500	.18805
	.14383	.04887	.00383	-.57482	-.02593	.00093	.04742	-.34589	3.42500	.19971
	.15368	.04966	.00400	-.57910	-.01702	.00079	.04173	-.34589	3.62500	.21137
	.16369	.05047	.00413	-.58170	-.00919	.00042	.03670	-.34589	3.82500	.22303
	.17387	.05130	.00416	-.58284	-.00230	-.00021	.03230	-.34589	4.02500	.23469
	.18421	.05213	.00403	-.58266	.00376	-.00112	.02845	-.34589	4.22500	.24636
	.19472	.05290	.00369	-.58138	.00911	-.00229	.02508	-.34589	4.42500	.25802
	.20537	.05358	.00310	-.57908	.01382	-.00368	.02210	-.34589	4.62500	.26968
	.21614	.05412	.00221	-.57569	.01797	-.00525	.01944	-.34589	4.82500	.28134
	.22700	.05444	.00099	-.57192	.02161	-.00693	.01702	-.34589	5.02500	.29300
	.23790	.05449	-.00057	-.56728	.02479	-.00868	.01475	-.34589	5.22500	.30466
	.24877	.05419	-.00248	-.56204	.02752	-.01042	.01259	-.34589	5.42500	.31633
	.25955	.05348	-.00474	-.55630	.02983	-.01210	.01048	-.34589	5.62500	.32799
	.27013	.05228	-.00731	-.55013	.03171	-.01365	.00836	-.34589	5.82500	.33965
	.28042	.05053	-.01019	-.54364	.03317	-.01503	.00620	-.34589	6.02500	.35131
	.29031	.04819	-.01331	-.53696	.03419	-.01616	.00397	-.34589	6.22500	.36297
	.29965	.04519	-.01663	-.52990	.03475	-.01701	.00166	-.34589	6.42500	.37464
	.30834	.04152	-.02009	-.52303	.03485	-.01754	-.00074	-.34589	6.62500	.38630
	.31622	.03715	-.02362	-.51609	.03445	-.01769	-.00323	-.34589	6.82500	.39796
	.32315	.03208	-.02714	-.50928	.03355	-.01745	-.00580	-.34589	7.02500	.40962
	.32900	.02630	-.03057	-.50271	.03212	-.01676	-.00844	-.34589	7.22500	.42128
	.33363	.01986	-.03382	-.49647	.03017	-.01566	-.01111	-.34589	7.42500	.43294
	.33690	.01279	-.03681	-.49067	.02768	-.01410	-.01378	-.34589	7.62500	.44461

Table 5 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.33871	.00516	-.03943	-.48543	.02466	-.01209	-.01641	-.34589	7.82500	.45627
.33893	-.00295	-.04161	-.48084	.02112	-.00964	-.01896	-.34589	8.02500	.46793
.33753	-.01145	-.04326	-.47702	.01709	-.00675	-.02137	-.34589	8.22500	.47959
.33434	-.02022	-.04429	-.47404	.01259	-.00347	-.02359	-.34589	8.42500	.49125
.32940	-.02912	-.04463	-.47201	.00767	.00017	-.02556	-.34589	8.62500	.50292
.32269	-.03802	-.04420	-.47100	.00238	.00414	-.02722	-.34589	8.82500	.51458
.31421	-.04875	-.04295	-.47107	-.00320	.00838	-.02852	-.34589	9.02500	.52624
.30401	-.05514	-.04083	-.47229	-.00900	.01284	-.02940	-.34589	9.22500	.53790
.29219	-.06302	-.03780	-.47468	-.01492	.01746	-.02980	-.34589	9.42500	.54956
.27885	-.07020	-.03384	-.47826	-.02088	.02216	-.02967	-.34589	9.62500	.56122
.26417	-.07649	-.02894	-.48303	-.02675	.02687	-.02898	-.34589	9.82500	.57289
.24833	-.08171	-.02310	-.48895	-.03243	.03152	-.02768	-.34589	10.02500	.58455
.23156	-.08567	-.01634	-.49598	-.03778	.03603	-.02575	-.34589	10.22500	.59621
.21415	-.08819	-.00871	-.50403	-.04268	.04031	-.02316	-.34589	10.42500	.60787
.19840	-.08910	-.00024	-.51301	-.04700	.04428	-.01990	-.34589	10.62500	.61953
.17863	-.08824	.00898	-.52279	-.05060	.04785	-.01598	-.34589	10.82500	.63120
.16123	-.08546	.01887	-.53320	-.05335	.05094	-.01141	-.34589	11.02500	.64286
.14458	-.08065	.02932	-.54406	-.05512	.05346	-.00622	-.34589	11.22500	.65452
.12911	-.07370	.04021	-.55517	-.05580	.05532	-.00043	-.34589	11.42500	.66618
.11825	-.06455	.05140	-.56630	-.05526	.05642	.00590	-.34589	11.62500	.67784
.10344	-.05314	.06272	-.57719	-.05340	.05669	.01272	-.34589	11.82500	.68950
.09414	-.03946	.07401	-.58757	-.05014	.05602	.01994	-.34589	12.02500	.70117
.09227	-.03569	.07680	-.59005	-.04910	.05570	.02180	-.34589	12.07500	.70408
.08574	-.01923	.08776	-.59938	-.04398	.05375	.02941	-.34589	12.27500	.71574
.08472	-.00662	.09823	-.60754	-.03732	.05068	.03722	-.34589	12.47500	.72741
.08662	.02001	.10796	-.61421	-.02909	.04642	.04512	-.34589	12.67500	.73907
.09285	.04250	.11671	-.61907	-.01928	.04087	.05297	-.34589	12.87500	.75073
.10373	.06661	.12422	-.62182	-.00791	.03398	.06064	-.34589	13.07500	.76239
.11958	.09208	.13021	-.62214	.00496	.02568	.06799	-.34589	13.27500	.77405
.14063	.11858	.13439	-.61974	.01925	.01590	.07485	-.34589	13.47500	.78571
.18706	.14570	.13647	-.61435	.03485	.00460	.08106	-.34589	13.67500	.79738
.19893	.17300	.13613	-.60572	.05162	-.00826	.08645	-.34589	13.87500	.80904
.23623	.19997	.13306	-.59364	.06936	-.02271	.09063	-.34589	14.07500	.82070
.27385	.22602	.12693	-.57792	.08787	-.03878	.09404	-.34589	14.27500	.83236
.32654	.25052	.11744	-.55845	.10689	-.05647	.09589	-.34589	14.47500	.84402
.37691	.27275	.10424	-.53515	.12613	-.07576	.09622	-.34589	14.67500	.85569
.43644	.29195	.08703	-.50801	.14527	-.09662	.09489	-.34589	14.87500	.86735
.49543	.30727	.06549	-.47708	.16396	-.11900	.09178	-.34589	15.07500	.87901
.55893	.31783	.03934	-.44246	.18185	-.14278	.08681	-.34589	15.27500	.89067
.6221[illegible]	.32268	.00830	-.40441	.19856	-.16781	.07998	-.34589	15.47500	.90233
.67656	.32197	-.01833	-.37375	.21010	-.18726	.07368	-.34589	15.62500	.91108
.70256	.31917	-.03771	-.35238	.21723	-.20048	.06895	-.34589	15.72500	.91691
.73435	.31438	-.05843	-.33032	.22388	-.21383	.06383	-.34589	15.82500	.92274
.76540	.30744	-.08048	-.30762	.22999	-.22727	.05837	-.34589	15.92500	.92857
.79576	.29824	-.10388	-.28434	.23554	-.24069	.05262	-.34589	16.02500	.93440
.82503	.28662	-.12862	-.26054	.24050	-.25403	.04664	-.34589	16.12500	.94023
.85300	.27247	-.15468	-.23626	.24486	-.26716	.04051	-.34589	16.22500	.94606

Table 5 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.87943	.25564	-.18204	-.21158	.24861	-.27996	.03434	-.34589	16.32500	.95190
.90494	.23602	-.21066	-.18656	.25173	-.29228	.02822	-.34589	16.42500	.95773
.92554	.21347	-.24047	-.16126	.25426	-.30392	.02229	-.34589	16.52500	.96356
.94663	.18789	-.27141	-.13573	.25620	-.31470	.01670	-.34589	16.62500	.96939
.96401	.15916	-.30338	-.11003	.25761	-.32436	.01160	-.34589	16.72500	.97522
.97836	.12718	-.33624	-.08422	.25854	-.33263	.00718	-.34589	16.82500	.98105
.98934	.09188	-.36984	-.05834	.25908	-.33920	.00363	-.34589	16.92500	.98688
.99662	.05320	-.40401	-.03242	.25931	-.34372	.00118	-.34589	17.02500	.99271
.99986	.01107	-.43851	-.00648	.25936	-.34580	.00005	-.34589	17.12500	.99854

TABLE 6

S = 17.15

System 3

	F	FP	FPP	G	GP	FPPP	GPP	H2	ETA POS FRACT	Z/S
WALL	.00000	.00000	.11792	.00000	.02131	.21935	.00000	-.10967		
DISK	.00000	.00000	-.22321	.00000	.02131	.21935	.00000	-.10967	.00000	.00000
	-.00108	-.02123	-.20146	.00206	.01925	.21613	-.03940	-.10967	.10000	.00583
	-.00417	-.04030	-.17993	.00374	.01375	.21464	-.06889	-.10967	.20000	.01166
	-.00907	-.05722	-.15850	.00473	.00577	.21430	-.08934	-.10967	.30000	.01749
	-.01555	-.07200	-.13706	.00484	-.00384	.21452	-.10167	-.10967	.40000	.02332
	-.02340	-.08463	-.11559	.00393	-.01433	.21470	-.10691	-.10967	.50000	.02915
	-.03240	-.09512	-.09414	.00196	-.02502	.21426	-.10613	-.10967	.60000	.03499
	-.04235	-.10348	-.07278	-.00106	-.03539	.21270	-.10045	-.10967	.70000	.04082
	-.05302	-.10968	-.05165	-.00509	-.04499	.20960	-.09096	-.10967	.80000	.04665
	-.06421	-.11380	-.03092	-.01002	-.05349	.20467	-.07873	-.10967	.90000	.05248
	-.07572	-.11568	-.01078	-.01574	-.06067	.19770	-.06473	-.10967	1.00000	.05831
	-.08733	-.11599	.00855	-.02211	-.06641	.18865	-.04982	-.10967	1.10000	.06414
	-.09885	-.11421	.02688	-.02897	-.07063	.17755	-.03474	-.10967	1.20000	.06997
	-.11011	-.11065	.04400	-.03619	-.07337	.16456	-.02006	-.10967	1.30000	.07580
	-.12093	-.10545	.05974	-.04360	-.07467	.14991	-.00624	-.10967	1.40000	.08163
	-.13115	-.09876	.07394	-.05108	-.07465	.13391	.00641	-.10967	1.50000	.08746
	-.14287	-.08852	.08935	-.06032	-.07296	.11251	.02029	-.10967	1.62500	.09475
	-.15865	-.06864	.10826	-.07438	-.06706	.07669	.03777	-.10967	1.82500	.10641
	-.17012	-.04569	.12005	-.08695	-.05822	.04127	.04972	-.10967	2.02500	.11808
	-.17882	-.02108	.12497	-.09754	-.04748	.00849	.05706	-.10967	2.22500	.12974
	-.17853	.00389	.12373	-.10587	-.03564	-.02011	.06085	-.10967	2.42500	.14140
	-.17531	.02807	.11726	-.11177	-.02331	-.04368	.06208	-.10967	2.62500	.15306
	-.16742	.05051	.10661	-.11519	-.01092	-.06200	.06153	-.10967	2.82500	.16472
	-.15527	.07050	.09280	-.11615	.00122	-.07525	.05969	-.10967	3.02500	.17638
	-.13942	.08749	.07682	-.11473	.01289	-.08391	.05667	-.10967	3.22500	.18805
	-.12050	.10114	.05950	-.11104	.02391	-.08862	.05324	-.10967	3.42500	.19971
	-.09920	.11125	.04159	-.10522	.03413	-.09003	.04887	-.10967	3.62500	.21137
	-.07624	.11777	.02367	-.09745	.04341	-.08876	.04378	-.10967	3.82500	.22303
	-.05233	.12075	.00623	-.08793	.05160	-.08534	.03803	-.10967	4.02500	.23469
	-.02817	.12032	-.01035	-.07689	.05858	-.08022	.03165	-.10967	4.22500	.24636
	-.00441	.11669	-.02577	-.06459	.06423	-.07375	.02473	-.10967	4.42500	.25802
	.01831	.11011	-.03978	-.05130	.06844	-.06618	.04737	-.10967	4.62500	.26968
	.03945	.10088	-.05218	-.03731	.07115	-.05770	.00970	-.10967	4.82500	.28134
	.05851	.08935	-.06281	-.02294	.07231	-.04845	.00187	-.10967	5.02500	.29300
	.07508	.07589	-.07152	-.00849	.07190	-.03856	-.00597	-.10967	5.22500	.30466
	.08876	.06088	-.07819	.00572	.06994	-.02812	-.01362	-.10967	5.42500	.31633
	.09934	.04475	-.08274	.01938	.06648	-.01727	-.02093	-.10967	5.62500	.32799

Table 6 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.10862	.02793	-.08508	.03221	.06160	-.00616	-.02771	-.10967	5.82500	.33965
.11050	.01087	-.08520	.04394	.05544	.00502	-.03380	-.10967	6.02500	.35131
.11098	-.00600	-.08309	.05431	.04814	.01601	-.03904	-.10967	6.22500	.36297
.10814	-.02222	-.07882	.06313	.03989	.02652	-.04329	-.10967	6.42500	.37464
.10216	-.03739	-.07253	.07022	.03090	.03621	-.04644	-.10967	6.62500	.38630
.09328	-.05111	-.06442	.07545	.02139	.04469	-.04838	-.10967	6.82500	.39796
.08183	-.06306	-.05476	.07876	.01163	.05159	-.04908	-.10967	7.02500	.40962
.06820	-.07294	-.04392	.08010	.00185	.05649	-.04850	-.10967	7.22500	.42128
.05281	-.08057	-.03232	.07951	-.00769	.05906	-.04667	-.10967	7.42500	.43294
.03812	-.08585	-.02047	.07706	-.01674	.05901	-.04367	-.10967	7.62500	.44461
.01862	-.08878	-.00890	.07286	-.02509	.05616	-.03962	-.10967	7.82500	.45627
.00076	-.08947	.00181	.06709	-.03253	.05046	-.03470	-.10967	8.02500	.46793
-.01703	-.08815	.01110	.05992	-.03892	.04203	-.02914	-.10967	8.22500	.47959
-.03439	-.08516	.01846	.05159	-.04416	.03119	-.02322	-.10967	8.42500	.49125
-.05191	-.08093	.02344	.04234	-.04821	.01844	-.01724	-.10967	8.62500	.50292
-.06671	-.07596	.02575	.03239	-.05107	.00449	-.01153	-.10967	8.82500	.51458
-.08138	-.07082	.02522	.02198	-.05286	-.00977	-.00642	-.10967	9.02500	.52624
-.09596	-.06606	.02189	.01131	-.05371	-.02328	-.00223	-.10967	9.22500	.53790
-.10787	-.06223	.01603	.00055	-.05383	-.03491	.00080	-.10967	9.42500	.54956
-.12005	-.05978	.00813	-.01019	-.05348	-.04344	.00248	-.10967	9.62500	.56122
-.13190	-.05906	-.00106	-.02083	-.05293	-.04771	.00273	-.10967	9.82500	.57289
-.14380	-.06023	-.01059	-.03137	-.05247	-.04662	.00161	-.10967	10.02500	.58455
-.15611	-.06324	-.01929	-.04184	-.05236	-.03924	-.00069	-.10967	10.22500	.59621
-.16920	-.06780	-.02582	-.05235	-.05280	-.02488	-.00382	-.10967	10.42500	.60787
-.18330	-.07333	-.02874	-.06301	-.05391	-.00311	-.00726	-.10967	10.62500	.61953
-.19854	-.07896	-.02657	-.07396	-.05568	.02613	-.01035	-.10967	10.82500	.63120
-.21481	-.08352	-.01781	-.08532	-.05798	.06260	-.01231	-.10967	11.02500	.64286
-.23178	-.08555	-.00109	-.09716	-.06048	.10566	-.01225	-.10967	11.22500	.65452
-.24875	-.08334	.02483	-.10949	-.06268	.15431	-.00924	-.10967	11.42500	.66618
-.26478	-.07494	.06092	-.12217	-.06391	.20722	-.00232	-.10967	11.62500	.67784
-.27818	-.05825	.10789	-.13493	-.06329	.26274	.00940	-.10967	11.82500	.68950
-.28730	-.03104	.16606	-.14729	-.05978	.31893	.02670	-.10967	12.02500	.70117
-.28864	-.02233	.18236	-.15025	-.05831	.16634	.03198	-.10967	12.07500	.70408
-.28931	.01597	.19413	-.16113	-.04976	-.03058	.05315	-.10967	12.27500	.71574
-.28232	.05336	.17618	-.16989	-.03738	-.13650	.06954	-.10967	12.47500	.72741
-.26832	.08548	.14358	-.17591	-.02242	-.18152	.07886	-.10967	12.67500	.73907
-.24860	.11047	.10607	-.17879	-.00631	-.18888	.08108	-.10967	12.87500	.75073
-.22464	.12797	.06940	-.17844	.00962	-.17544	.07740	-.10967	13.07500	.76239
-.19788	.13848	.03651	-.17502	.02436	-.15259	.06942	-.10967	13.27500	.77405
-.18965	.14290	.00851	-.16883	.03721	-.12739	.05875	-.10967	13.47500	.78571
-.14106	.14222	-.01456	-.16029	.04777	-.10374	.04673	-.10967	13.67500	.79738
-.11304	.13737	-.03320	-.14988	.05588	-.08335	.03439	-.10967	13.87500	.80904
-.08633	.12918	-.04813	-.13816	.06155	-.06652	.02245	-.10967	14.07500	.82070
-.06154	.11832	-.06002	-.12541	.06492	-.05280	.01136	-.10967	14.27500	.83236
-.03915	.10534	-.06940	-.11227	.06617	-.04128	.00139	-.10967	14.47500	.84402
-.01952	.09071	-.07661	-.09907	.06556	-.03096	-.00729	-.10967	14.67500	.85569
-.00295	.07483	-.08160	-.08616	.06335	-.02081	-.01460	-.10967	14.87500	.86735
.01036	.05813	-.08488	-.07382	.05982	-.00987	-.02045	-.10967	15.07500	.87901
.02028	.04103	-.08563	-.06230	.05527	.00275	-.02476	-.10967	15.27500	.89067
.02678	.02406	-.08362	-.05176	.05002	.01779	-.02743	-.10967	15.47500	.90233
.02946	.01176	-.07998	-.04457	.04583	.03107	-.02831	-.10967	15.62500	.91108
.03024	.00394	-.07639	-.04013	.04299	.04100	-.02833	-.10967	15.72500	.91691
.03026	-.00348	-.07175	-.03597	.04018	.05184	-.02791	-.10967	15.82500	.92274

Table 6 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.02956	-.01038	-.06599	-.03209	.03743	.06362	-.02704	-.10967	15.92500	.92857
.02821	-.01664	-.05900	-.02848	.03479	.07633	-.02572	-.10967	16.02500	.93440
.02826	-.02213	-.05069	-.02513	.03230	.08995	-.02398	-.10967	16.12500	.94023
.02331	-.02673	-.04098	-.02201	.03000	.10439	-.02185	-.10967	16.22500	.94606
.02095	-.03028	-.02979	-.01912	.02794	.11953	-.01937	-.10967	16.32500	.95190
.01780	-.03263	-.01706	-.01642	.02614	.13516	-.01661	-.10967	16.42500	.95773
.01447	-.03364	-.00275	-.01388	.02462	.15100	-.01366	-.10967	16.52500	.96356
.01112	-.03313	.01314	-.01148	.02341	.16665	-.01062	-.10967	16.62500	.96939
.00790	-.03096	.03056	-.00919	.02250	.18163	-.00764	-.10967	16.72500	.97522
.00499	-.02697	.04942	-.00697	.02187	.19526	-.00489	-.10967	16.82500	.98105
.00257	-.02103	.06954	-.00481	.02151	.20675	-.00255	-.10967	16.92500	.98698
.00085	-.01303	.09066	-.00266	.02134	.21510	-.00085	-.10967	17.02500	.99271
.00004	-.00288	.11243	-.00053	.02131	.21916	-.00004	-.10967	17.12500	.99854

TABLE 7

S = 30

System 1

VELOCITY AND SHEAR STRESS PROFILES

I	POSITION RELATIVE TO WALL Z/S	ETA / NUE	F / W	FP / GP	FPP / GPP
1	.00000000	.00000	.00000000	.00000000	-.16534420
		1.00000	.00000000	.00000000	.13566461
5	.00333333	.10000	-.00081034	-.01604317	-.15552212
		1.00000	.00162069	.01356628	.13565736
9	.00666667	.20000	-.00317594	-.03110559	-.14573501
		1.00000	.00635188	.02713007	.13560834
13	.01000000	.30000	-.00699894	-.04519241	-.13601504
		1.00000	.01399788	.04068532	.13548063
17	.01333333	.40000	-.01218217	-.05831182	-.12639154
		1.00000	.02436434	.05422251	.13524240
21	.01666667	.50000	-.01862942	-.07047483	-.11689120
		1.00000	.03725884	.06772921	.13486674
25	.02000000	.60000	-.02624570	-.08169497	-.10753822
		1.00000	.05249141	.08119053	.13433143
29	.02333333	.70000	-.03493751	-.09198811	-.09835452
		1.00000	.06987502	.09458957	.13361859
33	.02666667	.80000	-.04461302	-.10137218	-.08935990
		1.00000	.08922604	.10790786	.13271434
37	.03000000	.90000	-.05518230	-.10986699	-.08057219
		1.00000	.11036461	.12112572	.13160857
41	.03333333	1.00000	-.06655749	-.11749405	-.07200738
		1.00000	.13311499	.13422263	.13029452
45	.03666667	1.10000	-.07865296	-.12427638	-.06367974
		1.00000	.15730591	.14717755	.12876850
49	.04000000	1.20000	-.09138542	-.13023833	-.05560195
		1.00000	.18277085	.15996923	.12702956
53	.04333333	1.30000	-.10467412	-.13540547	-.04778517

Table 7 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
		1.00000	.20934825	.17257642	.12507918
57	.04666667	1.40000	-.11844091	-.13980439	-.04023916
		1.00000	.23688181	.18497813	.12292100
61	.05000000	1.50000	-.13261031	-.14346261	-.03297232
		1.00000	.26522063	.19715387	.12056056
65	.05666667	1.70000	-.16186921	-.14867071	-.01930352
		1.00000	.32373842	.22074865	.11526284
69	.06333333	1.90000	-.19190421	-.15126319	-.00682184
		1.00000	.38380842	.24321189	.10925878
73	.07000000	2.10000	-.22221612	-.15148033	.00444710
		1.00000	.44443225	.26441096	.10263703
77	.07666667	2.30000	-.25235423	-.14956589	.01449353
		1.00000	.50470846	.28423211	.09549668
81	.08333333	2.50000	-.28191666	-.14576414	.02332202
		1.00000	.56383333	.30258205	.08794263
85	.09000000	2.70000	-.31055021	-.14031713	.03095013
		1.00000	.62110043	.31938872	.08008189
89	.09666667	2.90000	-.33794967	-.13346219	.03740702
		1.00000	.67589933	.33460144	.07202059
93	.10333333	3.10000	-.36385661	-.12542979	.04273215
		1.00000	.72771323	.34819046	.06386157
97	.11000000	3.30000	-.38805789	-.11644157	.04697397
		1.00000	.77611577	.36014608	.05570254
101	.11666667	3.50000	-.41038363	-.10670870	.05018860
		1.00000	.82076725	.37047756	.04763460
105	.12333333	3.70000	-.43070504	-.09643045	.05243859
		1.00000	.86141007	.37921158	.03974123
109	.13000000	3.90000	-.44893190	-.08579307	.05379164
		1.00000	.89786378	.38639072	.03209755
113	.13666666	4.10000	-.46500984	-.07496882	.05431943
		1.00000	.93001968	.39207168	.02476978
117	.14333333	4.30000	-.47891751	-.06411535	.05409645
		1.00000	.95783501	.39632352	.01781510
121	.15000000	4.50000	-.49066357	-.05337521	.05319890
		1.00000	.98132713	.39922581	.01128150
125	.15666667	4.70000	-.50028366	-.04287563	.05170370
		1.00000	1.00056732	.40086681	.00520792
129	.16333333	4.90000	-.50783735	-.03272847	.04968750
		1.00000	1.01567470	.40134168	-.00037547
133	.17000000	5.10000	-.51340502	-.02303033	.04722587
		1.00000	1.02681004	.40075076	-.00544699
137	.17666667	5.30000	-.51708489	-.01386290	.04439248
		1.00000	1.03416978	.39919794	-.00999294
141	.18333333	5.50000	-.51899006	-.00529337	.04125842
		1.00000	1.03798012	.39678906	-.01400710
145	.19000000	5.70000	-.51924568	.00262496	.03789161
		1.00000	1.03849135	.39363052	-.01749018
149	.19666667	5.90000	-.51798619	.00985205	.03435629
		1.00000	1.03597237	.38982794	-.02044916
153	.20333333	6.10000	-.51535281	.01636027	.03071262
		1.00000	1.03070562	.38548496	-.02289669
157	.21000000	6.30000	-.51149110	.02213359	.02701628
		1.00000	1.02298220	.38070219	-.02485039
161	.21666667	6.50000	-.50654874	.02716668	.02331830

Table 7 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
		1.00000	1.01309748	.37557627	-.02633222
165	.23000000	6.90000	-.49401139	.03503834	.01609711
		1.00000	.98802277	.36465700	-.02798574
169	.24333333	7.30000	-.47889200	.04010909	.00935872
		1.00000	.95778399	.35339373	-.02809473
173	.25666667	7.70000	-.46226565	.04262080	.00333402
		1.00000	.92453130	.34235167	-.02692564
177	.27000000	8.10000	-.44509423	.04289252	-.00182129
		1.00000	.89018845	.33198654	-.02475655
181	.28333333	8.50000	-.42820142	.04129154	-.00602030
		1.00000	.85640284	.32264306	-.02186216
185	.29666667	8.90000	-.41225872	.03820756	-.00923638
		1.00000	.82451744	.31455863	-.01850174
189	.31000000	9.30000	-.39778104	.03403048	-.01149344
		1.00000	.79556207	.30787239	-.01490987
193	.32333333	9.70000	-.38513035	.02913225	-.01285565
		1.00000	.77026070	.30263451	-.01129003
197	.33666667	10.10000	-.37452588	.02385282	-.01341706
		1.00000	.74905176	.29842147	-.00781074
201	.35000000	10.50000	-.36605898	.01849020	-.01329177
		1.00000	.73211795	.29634941	-.00460412
205	.36333334	10.90000	-.35971127	.01329428	-.01260494
		1.00000	.71942253	.29508886	-.00176623
209	.37666667	11.30000	-.35537457	.00846399	-.01148493
		1.00000	.71074915	.29487885	.00064084
213	.39000000	11.70000	-.35287137	.00414740	-.01005665
		1.00000	.70574273	.29553981	.00258587
217	.40333334	12.10000	-.35197467	.00044422	-.00843635
		1.00000	.70394935	.29688527	.00406459
221	.41666666	12.50000	-.35242663	-.00258997	-.00672772
		1.00000	.70485325	.29873168	.00509533
225	.43000000	12.90000	-.35395503	-.00493806	-.00501931
		1.00000	.70791005	.30090661	.00571462
229	.44333334	13.30000	-.35628749	-.00661514	-.00338325
		1.00000	.71257499	.30325522	.00597307
233	.45666666	13.70000	-.35916300	-.00766180	-.00187519
		1.00000	.71832600	.30564506	.00593163
237	.47000000	14.10000	-.36234079	-.00813779	-.00053547
		1.00000	.72468157	.30796962	.00565862
241	.48333333	14.50000	-.36560687	-.00811633	.00060837
		1.00000	.73121373	.31015084	.00522741
245	.49666667	14.90000	-.36877847	-.00767972	.00153762
		1.00000	.73755694	.31214071	.00471499

POSITION RELATIVE TO DISK

I	Z/S	ETA NUE	F W	FP GP	FPP GPP
1	.00000000	.00000	.00000000	.00000000	.47249216
		1.00000	.00000000	1.00000000	-.52405271
5	.00333333	.10000	.00221649	.04291293	.38746522
		1.00000	-.00443299	.94774297	-.51969479
9	.00666667	.20000	.00831588	.07782247	.31231881

Table 7 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
		1.00000	-.01663175	.89630630	-.50796772
13	.01000000	.30000	.01754613	.10568406	.24639221
		1.00000	-.03509226	.84633572	-.49064209
17	.01333333	.40000	.02924741	.12738492	.18898529
		1.00000	-.05849482	.79831503	-.46919362
21	.01666667	.50000	.04284504	.14374124	.13938221
		1.00000	-.08569008	.75259345	-.44484472
25	.02000000	.60000	.05784240	.15549770	.09687001
		1.00000	-.11568481	.70940907	-.41860045
29	.02333333	.70000	.07381380	.16332838	.06075283
		1.00000	-.14762759	.66890907	-.39127984
33	.02666667	.80000	.09039749	.16783912	.03036254
		1.00000	-.18079498	.63116683	-.36354309
37	.03000000	.90000	.10728905	.16957065	.00506631
		1.00000	-.21457809	.59619669	-.33591545
41	.03333333	1.00000	.12423502	.16900238	-.01572841
		1.00000	-.24847004	.56396630	-.30880795
45	.03666667	1.10000	.14102700	.16655666	-.03257085
		1.00000	-.28205400	.53440718	-.28253546
49	.04000000	1.20000	.15749614	.16260309	-.04596561
		1.00000	-.31499229	.50742348	-.25733243
53	.04333333	1.30000	.17350812	.15746308	-.05637261
		1.00000	-.34701623	.48289940	-.23336641
57	.04666667	1.40000	.18895850	.15141425	-.06420802
		1.00000	-.37791701	.46070518	-.21074986
61	.05000000	1.50000	.20376865	.14469469	-.06984586
		1.00000	-.40753729	.44070216	-.18955015
65	.05666667	1.70000	.23126020	.13002285	-.07582774
		1.00000	-.46252041	.40669418	-.15149593
69	.06333333	1.90000	.25573608	.11471260	-.07655790
		1.00000	-.51147214	.37972341	-.11913826
73	.07000000	2.10000	.27716131	.09963366	-.07376474
		1.00000	-.55432262	.35868242	-.09212249
77	.07666667	2.30000	.29564330	.08535386	-.06875306
		1.00000	-.59128659	.34255240	-.06993490
81	.08333333	2.50000	.31137931	.07221558	-.06248542
		1.00000	-.62275861	.33042549	-.05199180
85	.09000000	2.70000	.32461774	.06039684	-.05565378
		1.00000	-.64923549	.32151245	-.03769808
89	.09666667	2.90000	.33563028	.04995919	-.04874018
		1.00000	-.67126057	.31514104	-.02648400
93	.10333333	3.10000	.34469240	.04088450	-.04206676
		1.00000	-.68938480	.31074855	-.01782637
97	.11000000	3.30000	.35207035	.03310279	-.03583607
		1.00000	-.70414070	.30787127	-.01125918
101	.11666667	3.50000	.35801299	.02651277	-.03016294
		1.00000	-.71602598	.30613267	-.00637700
105	.12333333	3.70000	.36274708	.02099687	-.02509946
		1.00000	-.72549415	.30523132	-.00283387
109	.13000000	3.90000	.36647512	.01643172	-.02065405
		1.00000	-.73295024	.30492932	-.00033947
113	.13666666	4.10000	.36937501	.01269539	-.01680596
		1.00000	-.73875001	.30504172	.00134631
117	.14333333	4.30000	.37160079	.00967210	-.01351606

Table 7 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
		1.00000	-.74320158	.30542707	.00241931
121	.15000000	4.50000	.37328424	.00725505	-.01073480
		1.00000	-.74656848	.30597917	.00303717
125	.15666667	4.70000	.37453678	.00534790	-.00840782
		1.00000	-.74907357	.30662001	.00332514
129	.16333333	4.90000	.37545168	.00386532	-.00647991
		1.00000	-.75090336	.30729382	.00338138
133	.17000000	5.10000	.37610623	.00273289	-.00489771
		1.00000	-.75221246	.30796221	.00328173
137	.17666667	5.30000	.37656389	.00188653	-.00361132
		1.00000	-.75312778	.30860000	.00308382
141	.18333333	5.50000	.37687626	.00127169	-.00257530
		1.00000	-.75375251	.30919208	.00283069
145	.19000000	5.70000	.37708492	.00084243	-.00174912
		1.00000	-.75416984	.30973071	.00255372
149	.19666667	5.90000	.37722304	.00056043	-.00109727
		1.00000	-.75444607	.31021348	.00227518
153	.20333333	6.10000	.37731678	.00039396	-.00058907
		1.00000	-.75463356	.31064170	.00201027
157	.21000000	6.30000	.37738657	.00031697	-.00019840
		1.00000	-.75477314	.31101915	.00176879
161	.21666667	6.50000	.37744811	.00030823	.00009669
		1.00000	-.75489622	.31135117	.00155647
165	.23000000	6.90000	.37759078	.00042986	.00046982
		1.00000	-.75518156	.31190376	.00122818
169	.24333333	7.30000	.37780593	.00065714	.00063954
		1.00000	-.75561187	.31235047	.00102507
173	.25666667	7.70000	.37812165	.00092385	.00067612
		1.00000	-.75624330	.31273817	.00092937
177	.27000000	8.10000	.37854436	.00118607	.00062241
		1.00000	-.75708872	.31310499	.00091636
181	.28333333	8.50000	.37906578	.00141307	.00050307
		1.00000	-.75813156	.31347878	.00095989
185	.29666667	8.90000	.37966701	.00158159	.00033153
		1.00000	-.75933401	.31387709	.00103487
189	.31000000	9.30000	.38032068	.00167232	.00011504
		1.00000	-.76064134	.31430779	.00111817
193	.32333333	9.70000	.38099221	.00166826	-.00014170
		1.00000	-.76198442	.31476992	.00118878
197	.33666667	10.10000	.38164061	.00155420	-.00043409
		1.00000	-.76328120	.31525457	.00122767
201	.35000000	10.50000	.38221914	.00131704	-.00075597
		1.00000	-.76443829	.31574553	.00121768
205	.36333334	10.90000	.38267645	.00094665	-.00109849
		1.00000	-.76535290	.31622016	.00114363
209	.37666667	11.30000	.38295788	.00043709	-.00144957
		1.00000	-.76591577	.31665014	.00099250
213	.39000000	11.70000	.38300748	-.00021207	-.00179355
		1.00000	-.76601496	.31700250	.00075401
217	.40333334	12.10000	.38277046	-.00099427	-.00211131
		1.00000	-.76554091	.31724076	.00042119
221	.41666666	12.50000	.38219626	-.00189466	-.00238053
		1.00000	-.76439252	.31732647	-.00000879
225	.43000000	12.90000	.38124216	-.00288891	-.00257630

Table 7 (continued)

	Z/S	ETA NUE	F W	FP GP	FPP GPP
		1.00000	-.76248433	.31722104	-.00053377
229	.44333334	13.30000	.37987719	-.00394236	-.00267201
		1.00000	-.75975438	.31688783	-.00114576
233	.45666666	13.70000	.37808640	-.00500955	-.00264048
		1.00000	-.75617279	.31629474	-.00183020
237	.47000000	14.10000	.37587517	-.00603426	-.00245539
		1.00000	-.75175035	.31541689	-.00256528
241	.48333333	14.50000	.37327346	-.00695019	-.00209297
		1.00000	-.74654692	.31423964	-.00332166
245	.49666667	14.90000	.37033950	-.00768237	-.00153384
		1.00000	-.74067899	.31276162	-.00406233

TABLE 8

S = 30

System 2

	F	FP	FPP	G	GP	FPPP	GPP	H1	ETA POS	FRACT Z/S
WALL	1.00000	.00000	-.46340	.00000	.26878	-.35530	.00000	-.35530		
DISK	.00000	.00000	.15173	.00000	-.31859	-.35530	.00000	-.35530	.00000	.00000
	.00070	.01350	.11927	-.03181	-.31719	-.29555	.02700	-.35530	.10000	.00333
	.00260	.02404	.09239	-.06336	-.31339	-.24326	.04831	-.35530	.20000	.00667
	.00543	.03214	.07038	-.09443	-.30768	-.19794	.06520	-.35530	.30000	.01000
	.00896	.03826	.05259	-.12464	-.30046	-.15905	.07866	-.35530	.40000	.01333
	.01303	.04278	.03838	-.15448	-.29204	-.12607	.08941	-.35530	.50000	.01667
	.01748	.04603	.02719	-.18322	-.28265	-.09849	.09801	-.35530	.60000	.02000
	.02220	.04830	.01852	-.21098	-.27250	-.07578	.10484	-.35530	.70000	.02333
	.02711	.04980	.01189	-.23770	-.26173	-.05740	.11019	-.35530	.80000	.02667
	.03214	.05073	.00691	-.26331	-.25050	-.04281	.11424	-.35530	.90000	.03000
	.03724	.05123	.00322	-.28779	-.23892	-.03150	.11715	-.35530	1.00000	.03333
	.04238	.05141	.00052	-.31109	-.22711	-.02295	.11900	-.35530	1.10000	.03667
	.04752	.05136	-.00145	-.33320	-.21515	-.01670	.11989	-.35530	1.20000	.04000
	.05264	.05114	-.00288	-.35412	-.20316	-.01229	.11989	-.35530	1.30000	.04333
	.05774	.05079	-.00395	-.37384	-.19120	-.00935	.11906	-.35530	1.40000	.04667
	.06280	.05035	-.00479	-.39236	-.17937	-.00752	.11748	-.35530	1.50000	.05000
	.07270	.04926	-.00611	-.42592	-.15634	-.00604	.11238	-.35530	1.70000	.05667
	.08249	.04792	-.00730	-.45499	-.13456	-.00603	.10521	-.35530	1.90000	.06333
	.09191	.04633	-.00854	-.47985	-.11435	-.00634	.09663	-.35530	2.10000	.07000
	.10100	.04450	-.00982	-.50085	-.09596	-.00636	.08723	-.35530	2.30000	.07667
	.10970	.04241	-.01105	-.51836	-.07948	-.00586	.07753	-.35530	2.50000	.08333
	.11795	.04009	-.01213	-.53277	-.06494	-.00484	.06794	-.35530	2.70000	.09000
	.12572	.03757	-.01296	-.54446	-.05228	-.00344	.05878	-.35530	2.90000	.09667
	.13297	.03492	-.01349	-.55380	-.04139	-.00182	.05024	-.35530	3.10000	.10333
	.13968	.03220	-.01369	-.56113	-.03213	-.00016	.04245	-.35530	3.30000	.11000
	.14585	.02947	-.01356	-.56675	-.02435	.00141	.03548	-.35530	3.50000	.11667
	.15148	.02680	-.01314	-.57096	-.01789	.00277	.02933	-.35530	3.70000	.12333
	.15658	.02423	-.01247	-.57398	-.01257	.00388	.02399	-.35530	3.90000	.13000
	.16118	.02182	-.01160	-.57605	-.00824	.00471	.01940	-.35530	4.10000	.13667
	.16532	.01980	-.01060	-.57734	-.00476	.00525	.01551	-.35530	4.30000	.14333
	.16903	.01759	-.00952	-.57800	-.00199	.00552	.01225	-.35530	4.50000	.15000
	.17237	.01579	-.00841	-.57817	.00018	.00556	.00955	-.35530	4.70000	.15667
	.17536	.01422	-.00731	-.57796	.00186	.00540	.00734	-.35530	4.90000	.16333
	.17607	.01286	-.00626	-.57746	.00314	.00508	.00556	-.35530	5.10000	.17000
	.18052	.01171	-.00529	-.57673	.00411	.00484	.00413	-.35530	5.30000	.17667
	.18277	.01074	-.00441	-.57583	.00482	.00411	.00301	-.35530	5.50000	.18333
	.18483	.00994	-.00365	-.57481	.00533	.00352	.00214	-.35530	5.70000	.19000
	.18675	.00927	-.00301	-.57371	.00569	.00291	.00146	-.35530	5.90000	.19667
	.18855	.00873	-.00249	-.57255	.00592	.00229	.00095	-.35530	6.10000	.20333
	.19025	.00827	-.00209	-.57134	.00607	.00167	.00056	-.35530	6.30000	.21000
	.19186	.00788	-.00182	-.57012	.00615	.00108	.00026	-.35530	6.50000	.21667
	.19488	.00721	-.00160	-.56765	.00617	.00001	-.00016	-.35530	6.90000	.23000
	.19763	.00655	-.00178	-.56520	.00604	-.00087	-.00047	-.35530	7.30000	.24333
	.20009	.00574	-.00227	-.56283	.00579	-.00154	-.00079	-.35530	7.70000	.25667

Table 8 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.20219	.00470	-.00299	-.56059	.00540	-.00199	-.00115	-.35530	8.10000	.27000
.20081	.00334	-.00384	-.55853	.00486	-.00222	-.00159	-.35530	8.50000	.28333
.20481	.00162	-.00473	-.55673	.00412	-.00222	-.00211	-.35530	8.90000	.29667
.20506	-.00044	-.00559	-.55526	.00316	-.00201	-.00269	-.35530	9.30000	.31000
.20442	-.00283	-.00631	-.55423	.00197	-.00158	-.00328	-.35530	9.70000	.32333
.20276	-.00548	-.00683	-.55372	.00054	-.00097	-.00386	-.35530	10.10000	.33667
.20002	-.00825	-.00708	-.55383	-.00111	-.00017	-.00436	-.35530	10.50000	.35000
.19616	-.01107	-.00694	-.55463	-.00294	.00076	-.00475	-.35530	10.90000	.36333
.19119	-.01375	-.00642	-.55619	-.00489	.00185	-.00497	-.35530	11.30000	.37667
.18520	-.01814	-.00545	-.55855	-.00888	.00299	-.00496	-.35530	11.70000	.39000
.17834	-.01805	-.00402	-.56169	-.00882	.00416	-.00470	-.35530	12.10000	.40333
.17084	-.01930	-.00213	-.56558	-.01060	.00527	-.00414	-.35530	12.50000	.41667
.16301	-.01970	.00018	-.57013	-.01210	.00627	-.00328	-.35530	12.90000	.43000
.15522	-.01910	.00286	-.57520	-.01318	.00707	-.00209	-.35530	13.30000	.44333
.14788	-.01738	.00580	-.58060	-.01373	.00759	-.00061	-.35530	13.70000	.45667
.14448	-.01444	.00889	-.58810	-.01363	.00775	.00114	-.35530	14.10000	.47000
.13649	-.01027	.01195	-.59141	-.01279	.00747	.00309	-.35530	14.50000	.48333
.13342	-.00491	.01479	-.59623	-.01114	.00669	.00518	-.35530	14.90000	.49667
.13271	.00152	.01728	-.60021	-.00865	.00554	.00729	-.35530	15.30000	.51000
.13475	.00882	.01912	-.60303	-.00532	.00355	.00932	-.35530	15.70000	.52333
.13984	.01889	.02006	-.60436	-.00122	.00109	.01112	-.35530	16.10000	.53667
.14812	.02473	.01993	-.60392	.00353	-.00183	.01253	-.35530	16.50000	.55000
.15958	.03247	.01855	-.60148	.00874	-.00510	.01341	-.35530	16.90000	.56333
.17399	.03939	.01582	-.59690	.01417	-.00859	.01361	-.35530	17.30000	.57667
.19091	.04493	.01168	-.59015	.01952	-.01208	.01301	-.35530	17.70000	.59000
.20968	.04855	.00618	-.58134	.02446	-.01534	.01152	-.35530	18.10000	.60333
.22942	.04971	-.00053	-.57069	.02861	-.01809	.00909	-.35530	18.50000	.61667
.24908	.04800	-.00819	-.55860	.03160	-.02003	.00572	-.35530	18.90000	.63000
.26739	.04309	-.01641	-.54561	.03307	-.02086	.00149	-.35530	19.30000	.64333
.28309	.03488	-.02469	-.53239	.03270	-.02034	-.00346	-.35530	19.70000	.65667
.29485	.02340	-.03247	-.51973	.03024	-.01824	-.00889	-.35530	20.10000	.67000
.30143	.00905	-.03907	-.50849	.02557	-.01448	-.01448	-.35530	20.50000	.68333
.30178	-.00780	-.04383	-.49957	.01868	-.00905	-.01986	-.35530	20.90000	.69667
.29515	-.02588	-.04810	-.49382	.00977	-.00208	-.02456	-.35530	21.30000	.71000
.28119	-.04408	-.04532	-.49198	-.00082	.00619	-.02813	-.35530	21.70000	.72333
.28002	-.06147	-.04103	-.49462	-.01252	.01536	-.03008	-.35530	22.10000	.73667
.23234	-.07640	-.03297	-.50205	-.02460	.02497	-.02998	-.35530	22.50000	.75000
.19944	-.08733	-.02107	-.51424	-.03618	.03446	-.02749	-.35530	22.90000	.76333
.18321	-.09270	-.00556	-.53079	-.04624	.04320	-.02237	-.35530	23.30000	.77667
.13554	-.09238	.00833	-.54559	-.05215	.04687	-.01676	-.35530	23.60000	.78667
.11710	-.08971	.01844	-.55638	-.05506	.05207	-.01219	-.35530	23.80000	.79333
.09959	-.08498	.02912	-.56755	-.05698	.05469	-.00698	-.35530	24.00000	.80000
.08828	-.07803	.04027	-.57905	-.05781	.05666	-.00116	-.35530	24.20000	.80667
[illegible]	-.06883	.05174	-.59059	-.05741	.05788	.00521	-.35530	24.40000	.81333
.05588	-.05733	.06337	-.60193	-.05569	.05826	.01208	-.35530	24.60000	.82000
.04578	-.04349	.07498	-.61278	-.05255	.05772	.01938	-.35530	24.80000	.82667
.03864	-.02739	.08639	-.62285	-.04792	.05617	.02702	-.35530	25.00000	.83333
.03497	-.00890	.09738	-.63184	-.04173	.05352	.03490	-.35530	25.20000	.84000
.03519	.01156	.10772	-.63943	-.03395	.04970	.04291	-.35530	25.40000	.84667
.03972	.03407	.11717	-.64531	-.02456	.04481	.05094	-.35530	25.60000	.85333
.04894	.05855	.12548	-.64915	-.01358	.03820	.05865	-.35530	25.80000	.86000
.08317	.08418	.13236	-.65064	-.00104	.03039	.06650	-.35530	26.00000	.86667

Table 8 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.08268	.11118	.13754	-.64947	.01299	.02113	.07373	-.35530	26.20000	.87333
.10770	.13904	.14071	-.64535	.02841	.01034	.08038	-.35530	26.40000	.88000
.13833	.16731	.14157	-.63802	.04509	-.00200	.08828	-.35530	26.60000	.88667
.17462	.19550	.13980	-.62724	.06287	-.01593	.09126	-.35530	26.80000	.89333
.21649	.22304	.13509	-.61262	.08152	-.03149	.09513	-.35530	27.00000	.90000
.26375	.24931	.12710	-.59459	.10083	-.04868	.09773	-.35530	27.20000	.90667
.31608	.27364	.11551	-.57246	.12052	-.06749	.09887	-.35530	27.40000	.91333
.37302	.29526	.10000	-.54638	.14027	-.08792	.09840	-.35530	27.60000	.92000
.43395	.31335	.08024	-.51637	.15976	-.10990	.09820	-.35530	27.80000	.92667
.49807	.32705	.05593	-.48251	.17863	-.13337	.09215	-.35530	28.00000	.93333
.58441	.33541	.02880	-.44498	.19850	-.15819	.08623	-.35530	28.20000	.94000
.63181	.33743	-.00742	-.40401	.21299	-.18418	.07844	-.35530	28.40000	.94667
.68224	.33417	-.03655	-.37120	.22425	-.20426	.07144	-.35530	28.55000	.95167
.71544	.32947	-.05765	-.34843	.23114	-.21785	.06828	-.35530	28.65000	.95500
.74806	.32259	-.08012	-.32499	.23749	-.23152	.06076	-.35530	28.75000	.95833
.77988	.31340	-.10396	-.30095	.24328	-.24519	.05494	-.35530	28.85000	.96167
.81066	.30170	-.12916	-.27635	.24847	-.25879	.04889	-.35530	28.95000	.96500
.84014	.28753	-.15571	-.25127	.25305	-.27221	.04267	-.35530	29.05000	.96833
.86807	.27057	-.18359	-.22576	.25700	-.28531	.03638	-.35530	29.15000	.97167
.89416	.25076	-.21275	-.19969	.26033	-.29795	.03014	-.35530	29.25000	.97500
.91812	.22790	-.24318	-.17372	.26304	-.30995	.02405	-.35530	29.35000	.97833
.93905	.20209	-.27472	-.14731	.26515	-.32111	.01827	-.35530	29.45000	.98167
.95844	.17300	-.30734	-.12071	.26671	-.33121	.01296	-.35530	29.55000	.98500
.97414	.14059	-.34091	-.09398	.26776	-.33996	.00830	-.35530	29.65000	.98833
.98644	.10479	-.37528	-.06717	.26839	-.34706	.00447	-.35530	29.75000	.99167
.99499	.08552	-.41026	-.04031	.26869	-.35218	.00170	-.35530	29.85000	.99500
.99943	.02273	-.44564	-.01344	.26877	-.35494	.00020	-.35530	29.95000	.99833

TABLE 9

S = 30

System 3

	F	FP	FPP	G	GP	FPPP	GPP	H2	ETA POS	FRACT Z/S
WALL	.00000	.00000	.08651	.00000	.04098	.20628	.00000	-.10314		
DISK	.00000	.00000	-.23001	.00000	.04123	.20628	.00000	-.10314	.00000	.00000
	-.00112	-.02198	-.20975	.00405	.03909	.19934	-.04103	-.10314	.10000	.00333
	-.00433	-.04197	-.19007	.00770	.03333	.19469	-.07259	-.10314	.20000	.00667
	-.00944	-.06001	-.17078	.01063	.02486	.19185	-.09542	-.10314	.30000	.01000
	-.01627	-.07615	-.15166	.01261	.01451	.19030	-.11034	-.10314	.40000	.01333
	-.02461	-.09034	-.13267	.01349	.00303	.18949	-.11823	-.10314	.50000	.01667
	-.03427	-.10268	-.11375	.01320	-.00893	.18884	-.12004	-.10314	.60000	.02000
	-.04508	-.11310	-.09491	.01171	-.02081	.18785	-.11678	-.10314	.70000	.02333
	-.05683	-.12163	-.07621	.00905	-.03215	.18805	-.10947	-.10314	.80000	.02667
	-.06935	-.12835	-.05774	.00531	-.04260	.18304	-.09911	-.10314	.90000	.03000
	-.08244	-.13321	-.03965	.00057	-.05191	.17855	-.08864	-.10314	1.00000	.03333
	-.09593	-.13630	-.02209	-.00503	-.05989	.17240	-.07290	-.10314	1.10000	.03667
	-.10964	-.13785	-.00523	-.01136	-.06647	.16452	-.05884	-.10314	1.20000	.04000
	-.12340	-.13737	.01076	-.01828	-.07162	.15494	-.04447	-.10314	1.30000	.04333
	-.13768	-.13554	.02571	-.02564	-.07538	.14380	-.03086	-.10314	1.40000	.04667
	-.15046	-.13227	.03947	-.03331	-.07782	.13130	-.01819	-.10314	1.50000	.05000
	-.17526	-.12193	.06290	-.04908	-.07920	.10326	.00354	-.10314	1.70000	.05667
	-.19896	-.10747	.08064	-.06474	-.07677	.07317	.01991	-.10314	1.90000	.06333
	-.21875	-.09008	.09227	-.07961	-.07158	.04339	.03123	-.10314	2.10000	.07000
	-.23488	-.07094	.09815	-.09325	-.06456	.01587	.03834	-.10314	2.30000	.07667
	-.24709	-.05118	.09886	-.10536	-.05645	-.00801	.04226	-.10314	2.50000	.08333
	-.25536	-.03169	.09523	-.11579	-.04781	-.02752	.04390	-.10314	2.70000	.09000
	-.25954	-.01330	.08816	-.12447	-.03900	-.04242	.04399	-.10314	2.90000	.09667
	-.26079	.00341	.07856	-.13140	-.03028	-.05293	.04303	-.10314	3.10000	.10333
	-.25861	.01801	.06725	-.13660	-.02183	-.05951	.04133	-.10314	3.30000	.11000
	-.25375	.03025	.05498	-.14015	-.01378	-.06276	.03906	-.10314	3.50000	.11667
	-.24668	.03998	.04233	-.14215	-.00624	-.06336	.03630	-.10314	3.70000	.12333

Table 9 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
-.23792	.04718	.02977	-.14269	.00071	-.06194	.03311	-.10314	3.90000	.13000
-.22797	.05191	.01765	-.14191	.00698	-.05908	.02951	-.10314	4.10000	.13667
-.21732	.05429	.00620	-.13995	.01249	-.05528	.02554	-.10314	4.30000	.14333
-.20641	.05445	-.00443	-.13697	.01717	-.05094	.02124	-.10314	4.50000	.15000
-.19587	.05257	-.01416	-.13314	.02096	-.04637	.01666	-.10314	4.70000	.15667
-.18550	.04884	-.02298	-.12865	.02382	-.04179	.01187	-.10314	4.90000	.16333
-.17625	.04344	-.03089	-.12368	.02570	-.03734	.00694	-.10314	5.10000	.17000
-.16822	.03655	-.03793	-.11843	.02659	-.03311	.00194	-.10314	5.30000	.17667
-.16172	.02833	-.04415	-.11311	.02648	-.02914	-.00307	-.10314	5.50000	.18333
-.15697	.01894	-.04960	-.10791	.02537	-.02542	-.00802	-.10314	5.70000	.19000
-.15421	.00853	-.05433	-.10303	.02328	-.02192	-.01285	-.10314	5.90000	.19667
-.15369	-.00275	-.05838	-.09866	.02024	-.01859	-.01752	-.10314	6.10000	.20333
-.15536	-.01477	-.06177	-.09500	.01628	-.01537	-.02197	-.10314	6.30000	.21000
-.15957	-.02742	-.06453	-.09221	.01147	-.01219	-.02617	-.10314	6.50000	.21667
-.17581	-.05403	-.06812	-.08992	-.00054	-.00570	-.03365	-.10314	6.90000	.23000
-.20291	-.08155	-.06902	-.09300	-.01526	.00134	-.03971	-.10314	7.30000	.24333
-.24102	-.10885	-.06694	-.10241	-.03208	.00922	-.04408	-.10314	7.70000	.25667
-.28980	-.13466	-.06151	-.11865	-.05027	.01807	-.04652	-.10314	8.10000	.27000
-.34836	-.15757	-.05236	-.14270	-.06900	.02784	-.04674	-.10314	8.50000	.28333
-.41526	-.17601	-.03915	-.17400	-.08733	.03835	-.04448	-.10314	8.90000	.29667
-.48835	-.18831	-.02162	-.21238	-.10422	.04934	-.03948	-.10314	9.30000	.31000
-.56485	-.19271	.00034	-.25703	-.11852	.06048	-.03149	-.10314	9.70000	.32333
-.64123	-.18744	.02673	-.30688	-.12900	.07140	-.02034	-.10314	10.10000	.33667
-.71328	-.17076	.05738	-.35955	-.13436	.08168	-.00591	-.10314	10.50000	.35000
-.77607	-.14102	.09192	-.41331	-.13328	.09078	.01183	-.10314	10.90000	.36333
-.82416	-.09678	.12975	-.46514	-.12446	.09801	.03281	-.10314	11.30000	.37667
-.85143	-.03890	.16995	-.51166	-.10664	.10238	.05679	-.10314	11.70000	.39000
-.85150	.03931	.21110	-.54910	-.07868	.10258	.08337	-.10314	12.10000	.40333
-.81780	.13185	.25123	-.57315	-.03969	.09692	.11186	-.10314	12.50000	.41667
-.74396	.23979	.28759	-.57930	.01093	.08340	.14125	-.10314	12.90000	.43000
-.62420	.36095	.31661	-.56285	.07324	.05983	.17010	-.10314	13.30000	.44333
-.45394	.49131	.33384	-.51921	.14668	.02418	.19647	-.10314	13.70000	.45667
-.23049	.62576	.33415	-.44421	.22976	-.02496	.21785	-.10314	14.10000	.47000
.04613	.75584	.31204	-.33446	.31990	-.08777	.23125	-.10314	14.50000	.48333
.37230	.87171	.26232	-.18786	.41325	-.16254	.23328	-.10314	14.90000	.49667
.74005	.96177	.18343	-.00413	.50457	-.23676	.22055	-.10314	15.30000	.51000
1.13667	1.01387	.07115	.21466	.58736	-.32365	.19027	-.10314	15.70000	.52333
1.54424	1.01431	-.07396	.46363	.65410	-.39893	.13993	-.10314	16.10000	.53667
1.93964	.95126	-.24465	.73470	.69647	-.44910	.06839	-.10314	16.50000	.55000
2.29571	.81659	-.42789	1.01644	.70815	-.45914	-.02302	-.10314	16.90000	.56333
2.58541	.60976	-.60468	1.29429	.67597	-.41464	-.12493	-.10314	17.30000	.57667
2.77477	.33721	-.75089	1.55125	.60116	-.30507	-.24458	-.10314	17.70000	.59000
2.84675	.01674	-.83967	1.76912	.48074	-.12798	-.35585	-.10314	18.10000	.60333
2.78549	-.32844	-.84554	1.93027	.31676	.10863	-.44991	-.10314	18.50000	.61667
2.59030	-.64612	-.75007	2.01988	.12514	.37297	-.51156	-.10314	18.90000	.63000
2.27654	-.90926	-.54855	2.02826	-.08424	.62660	-.52648	-.10314	19.30000	.64333
1.87023	-1.07271	-.25599	1.95318	-.28839	.81610	-.48422	-.10314	19.70000	.65667
1.43569	-1.16693	.08921	1.80142	-.46353	.88246	-.38171	-.10314	20.10000	.67000
1.00933	-1.00229	.42685	1.58931	-.58670	.77433	-.22667	-.10314	20.50000	.68333
.65017	-.77636	.68322	1.34135	-.64056	.47688	-.03976	-.10314	20.90000	.69667
.39828	-.47616	.78702	1.08703	-.61864	.02142	.14585	-.10314	21.30000	.71000
.26962	-.17354	.69097	.85547	-.52954	-.50263	.28904	-.10314	21.70000	.72333
.24881	.04965	.39474	.66906	-.39840	-.95185	.35031	-.10314	22.10000	.73667
.28929	.12380	-.05702	.53727	-.25357	-1.14990	.30552	-.10314	22.50000	.75000

Table 9 (continued)

F	FP	FPP	G	GP	FPPP	GPP	HI	ETA	Z/S
.32382	.01980	-.46833	.45300	-.16745	-.92313	.16115	-.10314	22.90000	.76333
.28604	-.22452	-.70136	.39381	-.14158	-.14295	-.03359	-.10314	23.30000	.77667
.18746	-.42790	-.60574	.34780	-.17140	.83662	-.15676	-.10314	23.60000	.78667
.09113	-.52707	-.35740	.31003	-.20764	1.66955	-.19824	-.10314	23.80000	.79333
-.01690	-.55895	.07059	.26452	-.24710	1.87639	-.18581	-.10314	24.00000	.80000
-.12708	-.51332	.35766	.21176	-.27832	1.03566	-.12097	-.10314	24.20000	.80667
-.22143	-.42549	.50044	.15422	-.29426	.42831	-.03737	-.10314	24.40000	.81333
-.29611	-.31986	.54228	.09518	-.29343	.02000	.04414	-.10314	24.60000	.82000
-.34930	-.21289	.51905	.03786	-.27755	-.22914	.11174	-.10314	24.80000	.82667
-.38186	-.11470	.45857	-.01506	-.25001	-.35892	.16030	-.10314	25.00000	.83333
-.39613	-.03059	.38104	-.06163	-.21474	-.40521	.18926	-.10314	25.20000	.84000
-.39517	.03750	.30005	-.10069	-.17548	-.39786	.20077	-.10314	25.40000	.84667
-.38219	.08976	.22390	-.13177	-.13536	-.36006	.19836	-.10314	25.60000	.85333
-.38022	.12767	.15692	-.15494	-.09680	-.30646	.18588	-.10314	25.80000	.86000
-.33194	.15325	.10068	-.17071	-.06144	-.25414	.16694	-.10314	26.00000	.86667
-.29960	.16865	.05500	-.17980	-.03025	-.20386	.14454	-.10314	26.20000	.87333
-.26562	.17583	.01874	-.18312	-.00370	-.16029	.12096	-.10314	26.40000	.88000
-.22987	.17687	-.00966	-.18159	.01816	-.12503	.09782	-.10314	25.60000	.88667
-.19489	.17244	-.03179	-.17615	.03552	-.09747	.07612	-.10314	26.80000	.89333
-.16094	.16426	-.04908	-.16766	.04874	-.07639	.05643	-.10314	27.00000	.90000
-.12918	.15305	-.06267	-.15690	.05625	-.06028	.03901	-.10314	27.20000	.90667
-.09990	.13940	-.07341	-.14458	.06450	-.04752	.02390	-.10314	27.40000	.91333
-.07355	.12384	-.08180	-.13129	.06796	-.03661	.01106	-.10314	27.60000	.92000
-.05045	.10682	-.08809	-.11755	.06907	-.02620	.00041	-.10314	27.80000	.92667
-.03089	.08875	-.09224	-.10379	.06827	-.01511	-.00812	-.10314	28.00000	.93333
-.01590	.07006	-.09402	-.09034	.06596	-.00229	-.01457	-.10314	28.20000	.94000
-.00286	.05133	-.09298	-.07747	.06258	.01311	-.01896	-.10314	28.40000	.94667
-.00380	.03758	-.09001	-.06831	.05957	.02679	-.02068	-.10314	28.55000	.95167
.00711	.02875	-.08683	-.06246	.05745	.03705	-.02150	-.10314	28.65000	.95500
.00956	.02025	-.08257	-.05682	.05529	.04828	-.02161	-.10314	28.75000	.95833
.01118	.01225	-.07714	-.05140	.05314	.06047	-.02122	-.10314	28.85000	.96167
.01203	.00486	-.07045	-.04619	.05106	.07363	-.02033	-.10314	28.95000	.96500
.01218	-.00179	-.06239	-.04118	.04909	.08767	-.01899	-.10314	29.05000	.96833
.01170	-.00757	-.05288	-.03637	.04728	.10250	-.01722	-.10314	29.15000	.97167
.01070	-.01232	-.04187	-.03172	.04566	.11793	-.01511	-.10314	29.25000	.97500
.00928	-.01589	-.02929	-.02723	.04427	.13370	-.01271	-.10314	29.35000	.97833
.00756	-.01812	-.01513	-.02286	.04312	.14943	-.01014	-.10314	29.45000	.98167
.00570	-.01888	.00058	-.01859	.04224	.16466	-.00753	-.10314	29.55000	.98500
.00385	-.01796	.01777	-.01440	.04162	.17876	-.00502	-.10314	29.65000	.98833
.00217	-.01526	.03627	-.01026	.04123	.19094	-.00281	-.10314	29.75000	.99167
.00086	-.01067	.05586	-.00615	.04104	.20025	-.00111	-.10314	29.85000	.99500
-.00016	-.00467	.07820	-.00205	.04098	.20555	-.00013	-.10314	29.95000	.99833

TABLE 10

<u>Different Order Contributions to Shear Stresses for S = 1.0</u>

DISK

r	rF''_o	$F''_{1/r}$	$F_2''/r^3 \times 10^6$	rG'_o	$G'_{1/r}$	$G_2'/r^3 \times 10^8$
100	10.041	.0599	- .1751	- 100.398	- .00599	.053
200	20.082	.0299	- .0218	- 200.796	- .00299	.007
300	30.123	.0199	- .0065	- 301.194	- .00199	.002
400	40.164	.0149	- .0027	- 401.59	- .0149	.0008
500	50.205	.0119	- .0014	- 501.99	- .00119	.0004
WALL						
100	6.622	- .0601	.1751	- 099.83	.00399	.053
200	13.242	- .0301	.0218	- 199.66	.00199	.007
300	19.866	- .020	.0065	- 299.69	.00133	.002
400	26.488	- .015	.0027	- 399.32	.00100	.0008
500	33.110	- .012	.0014	- 499.15	.00080	.0004

TABLE 11

Different Order Contributions to Shear Stresses for S = 17.15

DISK

r	rF_o''	F_1''/r	F_2''/r^3 X10^6	rG_o'	G_1'/r	G_2'/r^3 X10^8
100	46.893	.00141	- .2232	- 52.033	- .00316	2.131
200	93.786	.00070	- .0279	-104.065	- .00158	0.266
300	140.679	.00047	- .0083	-156.130	- .00105	0.0789
400	187.572	.00035	- .0035	-208.130	- .00789	0.0333
500	234.465	.00028	- .0018	-260.163	- .00063	0.0171

WALL

r	rF_o''	F_1''/r	F_2''/r^3 X10^6	rG_o'	G_1'/r	G_2'/r^3 X10^8
100	16.978	- .00447	.1179	- 13.93	.00259	2.131
200	33.956	- .00224	.01474	- 27.86	.00129	0.266
300	50.934	- .00149	.00437	- 41.79	.00086	0.0789
400	67.912	- .00112	.00184	- 55.72	.00065	0.0333
500	84.890	- .00089	.00094	- 69.65	.00052	0.0171

TABLE 12

Different Order Contributions to Shear Stresses for S = 30.0

r	rF''_o	F''_1/r	rG'_o	G'_1/r
DISK				
100	47.2492	.0015173	- 52.4053	- .003186
200	94.4984	.0007583	- 104.8105	- .001593
300	141.7476	.0005058	- 157.2158	- .001062
400	188.9968	.0003793	- 209.621	- .000797
500	236.242	.0003035	- 262.026	- .000637
WALL				
100	16.5344	-.004634	- 13.5665	.002688
200	33.0688	-.002317	- 27.1329	.001344
300	49.6033	-.001545	- 40.6994	.000896
400	66.1377	-.001159	- 54.2658	.000672
500	82.6721	-.000927	- 67.8323	.0005376

TABLE 13

Different Order Contributions to Pressure Distribution for S = 1.0

r	$\beta^2 r^2/2$	$H_1 \ln(r)$	H_2/r^2 X10^4
10	14.93	- 27.91	- 77.71
50	373.27	- 47.41	- 03.108
100	1,493.0	- 55.82	- 00.7771
200	5,972.3	- 64.22	- 00.0971
300	13,437.8	- 69.13	- 00.0864
400	23,889.5	- 72.62	- 00.0486
500	37,327.3	- 75.33	- 00.0311

TABLE 14

Different Order Contributions to Pressure Distribution for S = 17.15

r	$\beta^2 r^2/2$	$H_1 \ln(r)$	$H_2/r^2 \times 10^4$
10	5.09	- 0.7964	- 10.97
50	127.3	- 1.3531	- 00.4386
100	509.1	- 1.8326	- 00.1097
200	2,036.24	- 1.9729	- 00.0274
300	4,581.54	- 1.9729	- 00.0122
400	8,144.9	- 2.0724	- 00.0068
500	12,726.5	- 2.1495	- 00.00439

TABLE 15

Different Order Contributions to Pressure Distribution for S = 30.0

r	$\beta^2 r^2/2$	$H_1 \ln(r)$
10	4.9141	- 0.818105
50	122.853	- 1.389942
100	491.4113	- 1.63622
200	1,965.645	- 1.88249
300	4,422.701	- 2.02655
400	7,862.580	- 2.128767
500	12,285.281	- 2.20805

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